

CRATER STATISTICS AND EROSION

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R. J. Collins\*

B. G. Smith

\*University of Minnesota, Minneapolis, Minnesota, supported by NASA Contract NSR-24-005-047 to the 'Tycho' study group, Department of Electrical Engineering, University of Minnesota.

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ABSTRACT

A review is presented of the essential features of lunar cratering statistics and their interpretation in terms of the meteoroid impact hypothesis. The crater size distribution function is introduced. A simple model is used to discuss the concept of crater lifetime and the mechanisms for its limitation, with relevance to the existence of saturated surfaces and equilibrium distribution functions. Secondary cratering and its importance are considered. It is pointed out that several different theoretical models adequately match the measured distributions and that this argues against the exclusive justification of any one model. The conclusion is drawn that a non-equilibrium distribution function should contain information about the crater production process but that an equilibrium distribution may be sensitive primarily to local surface properties.

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## CRATER STATISTICS AND EROSION

### INTRODUCTION

Craters are the most abundant and easily recognizable of the moon's topographic features and are present in greater or lesser numbers on all parts of the surface. Because of its symmetry a crater may be described, at least at first sight, by a few easily measured parameters, diameter, depth, etc., and this suggests the possibility of using simple models to examine the processes responsible for cratering.

Two principal mechanisms have been invoked to explain crater formation. The impact hypothesis recognizes that in the absence of an atmosphere interplanetary debris will strike the moon's surface producing impact craters and attributes to this origin the majority of the visible craters. Proponents of the volcanic hypothesis argue that most of these features are of internal origin. The impact hypothesis has been supported by quantitative studies relating observed crater distributions to the measured meteoroid flux through experimental and theoretical cratering models. Evidence for volcanism is largely qualitative. The difficulty of judging the volcanic hypothesis lies in the absence of numerical predictions from a model for this complex process; quantitative support for it is mainly negative relying on the demonstration of inconsistencies in the application of impact theory. Because of this difficulty the tone and detail of this review will favor the impact hypothesis, but it may be noted that many of the general features of the argument, especially those concerning equilibrium distributions, are applicable equally to volcanism.

### CRATERING STATISTICS

Approaches to the analysis of crater measurements and distributions fall into three groups of which the first considers the spatial distribution of crater centers. In the absence of any strong reason to the contrary it may be expected that to a first approximation the time average of

the meteoroid flux will be distributed isotropically over the moon. A homogeneous moon should then display a random distribution of crater centers following a Poisson law, for which the probability  $P_n$  of finding  $n$  crater centers per unit area is given by:

$$P_n = e^{-\lambda} \cdot \frac{\lambda^n}{n!} \quad (1)$$

where  $\lambda$  is the mean value of  $n$ . Arthur [1954] and Lenham [1964] have examined regions of the moon unsuccessfully for deviations from such a distribution. Possible exceptions to a random distribution could be an east-west asymmetry induced by the motion of the moon through space, favoring the leading hemisphere, and an excess of craters on the equatorial belt caused by a concentration of meteoroids in the ecliptic. Gravitational focusing by the earth might also produce a slight effect. Evidence for asymmetries is inconclusive although Fielder [1965] claims to observe departures from a random distribution. Marcus [1966 c] has pointed out that inhomogeneities in the surface and the oblitative effect of large craters would induce a slight departure from the simple law (equation 1) thus arguing that the variations in crater density observed by Fielder [1965] do not contradict the essentially random nature of the underlying spatial distribution. A demonstration of severely non-random cratering would provide an important argument in favor of the volcanic hypothesis.

A second line of study pursued by Baldwin [1949] concerns the relation of crater diameter to depth. Illustrated in Figure 1 is a plot of diameter as a function of depth measured for craters ranging from terrestrial explosion pits to the largest lunar craters. The smooth form of the interpolating curve is indicative of the uniform scaling of soil and rock mechanical properties involved in crater production but by itself is not a conclusive argument for a particular cratering mechanism.

The third approach has stimulated the greatest interest and regards the size distribution of crater diameters as the important quantity, linking it within the impact hypothesis to the mass distribution in the meteoroid flux. A general discussion has been presented by Stanyukovich and Bronshten [1962a] and by Shoemaker [1962]. Experimental crater counts per unit area are conventionally presented in one of two forms: as a cumulative distribution for which the number,  $F(D)$ , of craters with diameter greater than  $D$  is displayed as a function of  $D$ , or as a differential distribution in which the number of craters in a size interval  $\Delta D$  about  $D$  is displayed as a density per unit interval,  $f(D)$ . For sufficiently small steps  $\Delta D$  the differential distribution is, as its name implies, minus the derivative of the cumulative. It is convenient to present distributions on diagrams with logarithmic scale and in this form they are found to be almost linear in sections. Empirical power law fits to the distribution functions are useful in theoretical manipulations:

$$F(D) = AD^{-\alpha} \quad (2)$$

and

$$f(D) = \alpha AD^{-\alpha-1} = BD^{-\beta}$$

where  $A$ ,  $B$ ,  $\alpha$  and  $\beta$  are parameters.  $\alpha$  and  $\beta$  are called "population indices". Experimental distributions have been published by various authors and Figure 2 illustrates those by Dodd et al. [1963] in Mare Imbrium and by Baldwin and Palm and Strom [Baldwin, 1964] in sample highland areas, which are typical of the two basic sets into which crater counts at earth-based telescopic resolution may be separated. The set obtained from highlands has larger amplitude and steeper slope than that from maria. High resolution photography from Ranger spacecraft, and more recently from Lunar Orbiter, has allowed counts to be extended to meter sized craters. At these smaller sizes the difference between marial and highland distributions is greatly reduced, as may be seen in Figure 3, taken from an analysis of Ranger photography by Shoemaker [1966]. The asymptotic similarity of these distributions is discussed later.

Other experimental distributions have been published by Brinkmann [1966], Hartmann [1964], McGillem and Miller [1962], Miller [1965], Opik [1960], Shoemaker et al. [1963] and Young [1933] for different parts of the moon's surface.

IMPACT THEORY

The essentials of an impact theory may be illustrated with the assistance of a simple theoretical model similar to those discussed by Collins [1965, 1966] and Walker [1966]. Craters of all sizes are produced by impacting meteoroids and are subsequently destroyed by oblitative or erosive mechanisms; the appearance of the surface at any time represents a balance between the opposing processes. Craters in the diameter range  $\Delta D$  about  $D$  are formed per unit area at a rate  $\gamma(D)\Delta D$  and are destroyed at a rate described by a characteristic lifetime  $\tau(D)$ . The net birth rate per unit area of craters of this size is then:

$$\frac{d}{dt} f(D) = \gamma(D) - \frac{f(D)}{\tau(D)} \quad (3)$$

The lifetime  $\tau$  has been introduced phenomenologically but can be given a precise analytical meaning in terms of the detailed erosive mechanism [Marcus, 1964]. If no craters are present at time  $t=0$  integration of equation 3 with respect to time gives:

$$f(D) = \gamma(D) \tau(D) \{1 - e^{-t/\tau(D)}\} \quad (4)$$

where both production and destruction have been assumed independent of time, a condition which can be relaxed without difficulty. Marcus [1964] has examined the general case of time dependence. For times much smaller than the lifetime the crater distribution depends only upon the productive process and the elapsed time:

$$t \ll \tau(D) \quad , \quad f(D) \approx \gamma(D) \cdot t \quad (5)$$

For long times the distribution is in equilibrium, is not a function of time and is dependent upon both the productive and destructive processes:

$$t \gg \tau(D) \quad , \quad f(D) \approx \gamma(D) \cdot \tau(D) \quad (6)$$

Cratering is a remarkably complex process but experiments examining the impact of hypervelocity projectiles into various materials [Gault et al., 1963; Stanyukovich and Bronshten, 1962b] indicate that an idealized model may be constructed which is adequate for the present purpose. With these experiments it is found that the mass ejected from a crater is approximately proportional to the kinetic energy of the impacting projectile in the range  $10^8 - 10^{11}$  ergs. Extrapolation of the curve to higher energies ( $10^{13} - 10^{21}$  ergs) provides a reasonable fit as well to the dependence of the mass ejected from explosion craters upon expended energy (if the depth of burial of the charge is suitably chosen), Figure 4. If the crude assumption is made that all meteoroids strike the moon with the same velocity (estimated for example by Gault et al. [1963] to be  $\sim 10 - 30$  km/sec), it follows that the diameter of a crater will be proportional to the cube root of projectile mass (m) or proportional to its linear dimension (x):

$$D \propto m^{1/3}, \quad D \propto x \quad (7)$$

Estimates of the present average meteoroid flux per unit mass range,  $\Gamma(m)$ , in the vicinity of the earth have been made and with the assumption that this is not too different from the flux at the moon the crater production rate is given by:

$$\gamma(D) \propto D^2 \Gamma(m) \quad (8)$$

Figure 5 illustrates a composite of estimates of the cumulative meteoroid flux (the integral of  $\Gamma(m)$ ) collected by Hawkins [1964].

Crater destruction may have several contributors. For example, a crater will vanish when a larger crater is superimposed upon it or when its walls have been demolished by the repeated production of smaller craters within its rim [Marcus, 1966 a; Walker, 1966]. Again a crater will vanish when it has been filled by material thrown into it from the excavation of adjacent craters. Some general conclusions can be drawn about these (and any other) erosive processes and the last mechanism will provide a focus for a model analysis. The approach will be that of Collins [1965] simplified slightly to appeal more readily to intuition.



The rate at which volume of material is ejected from a unit area of the surface by impact craters in the diameter range  $\Delta D'$  about  $D'$  is simple  $v D'^3 \gamma(D') \Delta D'$ , where  $v$  is a numerical factor dependent upon crater shape and equal to  $\pi/12$  for hemispherical craters. The total rate  $Q$  contributed by craters of all sizes is then:

$$Q = v \int_0^{D_{\max}} D'^3 \gamma(D') dD' \quad (9)$$

where the lower limit on the integral is yet to be chosen and the upper limit is the largest diameter present in  $\gamma(D)$ . Collins [1965] discusses the value of  $D_{\max}$  to be used. Since only a negligible fraction of ejecta is traveling fast enough to escape the moon entirely,  $Q$  is also the average rate at which material volume is falling back upon a unit area of the surface, constituting the burying flux. A crater of diameter  $D$  will be completely filled or at least severely reduced in size after a time  $\tau(D)$  where:

$$\tau(D) \sim \frac{4vD^3}{\pi D^2 Q} \quad (10)$$

If it is assumed that most of the ejecta from a crater are distributed within a diameter from the crater rim and if the only craters allowed to partake of the destruction of another crater, diameter  $D$ , are those lying wholly outside its rim the bulk of the flux burying  $D$  will be produced by craters with diameter  $D' \gtrsim D$ . This implies a lower limit for the integral in equation 9 leading to:

$$\frac{1}{\tau(D)} = \frac{\pi}{4D} \cdot \int_D^{D_{\max}} D'^3 \gamma(D') dD' \quad (11)$$

If  $\Gamma(m)$ , and hence  $\gamma(D)$ , may be represented crudely by a single population index function of the form:

$$\gamma(D) = CD^{-\epsilon} \quad (12)$$

equation 11 may be integrated to give:

$$\frac{1}{\tau(D)} = \frac{\pi C}{4D(4-\epsilon)} \left[ D_{\max}^{4-\epsilon} - D^{4-\epsilon} \right] \quad (13)$$

The behavior of the quantity in brackets as a function of  $D$  depends upon the relative importance of the lower and upper limits in the integral, equation 11, and this in turn depends critically upon the sign of  $4-\epsilon$ . The physical significance of the two distinct situations lies in whether a large number of small craters or a small number of large craters is primarily responsible for limiting crater lifetime. The choice depends upon the details of the destructive model and the exact form of  $\gamma(D)$ , which is known only imprecisely from measurements of the meteoroid flux or from marial crater counts at large diameter. Using the present model as a vehicle in the more general context Collins [1965] argues that large craters will dominate, and with this assumption the lifetime becomes:

$$\tau(D) \approx \frac{4D(4-\epsilon)}{\pi C} \cdot D_{\max}^{\epsilon-4} \quad (14)$$

leading, with equation 6, to an expression for the equilibrium distribution independent of the amplitude of the productive flux but retaining information about its functional form:

$$t \gg \tau(D) \quad , \quad f(D) \approx \frac{4(4-\epsilon)}{D_{\max}^3 \pi} \cdot \left( \frac{D_{\max}}{D} \right)^{\epsilon-1} \quad (15)$$

The distribution, calculated with the expression for  $\Gamma(m)$  quoted by Hawkins [1964], is compared in Figure 6 with crater counts from Ranger VII photographs made by Shoemaker [1965]. The agreement is satisfactory, but  $D_{\max}$  has been considered essentially as an adjustable parameter, chosen here to be 20 km.

It is possible that this simple model overestimates the importance of large craters. Marcus [1966a] and Walker [1966], considering crater superimposition as the main

destructive mechanism, deduce that craters themselves of size  $\sim D$  play the decisive role. A parallel conclusion here would lead to the choice of the second term in parenthesis in equation 13, giving for the lifetime:

$$\tau(D) \sim \frac{(\epsilon-4) \cdot 4}{\pi C} D^{\epsilon-3} \quad (16)$$

which in turn leads to an equilibrium distribution:

$$t \gg \tau(D) \quad , \quad f(D) \sim \frac{(\epsilon-4) \cdot 4}{\pi} D^{-3} \quad (17)$$

The essential feature of this result is that the equilibrium distribution, equation 17, is now determined solely by factors arising geometrically and is independent of the productive and destructive mechanisms (apart from the model dependent numerical factor  $\epsilon-4$ ). Although the model is simple and in many respects arbitrary, the conclusion stands that if the crater producing mechanism is itself the erosive mechanism and if the latter is not dominated by the largest craters, then the equilibrium distribution contains little information about the formative processes. Marcus [1964] has made this point clear with a more general argument. The corollary is that, within this model, a measured equilibrium distribution cannot be used to distinguish between the applicability of volcanic or impact hypotheses.

The conclusion expressed in equation 17 is pleasing on two grounds; it is simple, and the inverse cube distribution which may be deduced by dimensional argument satisfies one's intuition about the close packing of craters on a saturated surface. More concretely the crater counts of Shoemaker [1966] from Ranger VII, VIII and IX photographs, illustrated in Figure 3, indicate a saturation distribution of just this form in maria and highlands alike, with an amplitude close to that of equation 17. It is perhaps surprising that the two different theoretical models both predict distributions in reasonable agreement with experiment. This coincidence is, unfortunately, shared by other models as well.

The expressions for crater lifetime deduced here are monotonically increasing functions of crater diameter. For a surface of a given age the distribution of small craters

(satisfying the inequality preceding equations 15 or 17) will be in equilibrium, while that for large craters will not (equation 5). If attention is limited to the latter the age of the surface may be estimated from the amplitude of the measured distribution, a technique employed by Young [1933], Kreiter [1960] and Shoemaker and Hackman [1962].

So far only the primary cratering process has been considered. It is likely that many of the fragments ejected at meteoroid impact will themselves produce craters adding to the overall population. The secondary cratering process is exceedingly complicated and no adequate model has yet been offered to describe it. Fragments from a primary crater will be members of a mass and velocity distribution and the size of the secondary crater which any one produces will depend upon its mass, velocity and possibly direction. Gault et al. [1966] have described the experimental situation. Idealization of the process within a mathematical model is much more difficult for the comparatively slow moving ejecta than for the hyper-velocity meteoroids. Gault et al. [1963] have described experimental investigations of the linear size distribution of ejected fragments which they conclude may be approximated by a population index function  $g(x)$ :

$$g(x) = \frac{R_1}{x} \cdot \left( \frac{x}{x_m} \right)^\mu, \quad x \leq x_m \quad (18)$$

where  $g(x)\Delta x$  is the number of fragments in the size range  $\Delta x$  ejected from a crater of size  $D$ . The size of the largest fragment  $x_m$  is itself proportional to  $D$ ,  $x_m = \lambda D$ . The constants  $\mu$  ( $\sim -2.5$ ),  $R_1$ , and  $\lambda$  contain the numerical details of the physical mechanism.

If the patently incorrect simplifications are made that all fragments have a single characteristic velocity and that an impact produces a crater with a diameter proportional to the size of the fragment,  $D = \beta x$ , the total production rate of craters of diameter  $D$  may be written:

$$\gamma_{TOT}(D) = \gamma(D) + R_2 \cdot \int_{D/\beta\lambda}^{D_{max}} \frac{\gamma(D')}{D} \left( \frac{D}{D'} \right)^\mu d D' \quad (19)$$

where the range of integration extends over all primary craters large enough to produce secondary craters of size D. With  $\gamma(D)$  given by equation 12 the integral may be evaluated:

$$\gamma_{TOT}(D) = \gamma(D) \left\{ 1 + P \left| 1 - \left( \frac{\beta \lambda D_{max}}{D} \right)^{1-\epsilon-\mu} \right| \right\} \quad (20)$$

$R_2$  and P are numerical factors, and  $P \gtrsim 1$ .

The behavior of the quantity in brackets depends critically upon the sign of  $1-\epsilon-\mu$ . If this is negative the total distribution of craters much smaller than the maximum size secondary has the form of the primary distribution with an enhanced amplitude, Figure 7b. If  $1-\epsilon-\mu$  is positive, the form of the secondary distribution will dominate the primary for sufficiently small craters, rising more steeply with decreasing diameter, Figure 7a. In either case secondary craters should provide a dominant contribution to the total distribution below a critical diameter estimated by Walker [1966] to be  $\sim 100$  meters. The particular criterion developed here for the relative importance of primary and secondary cratering is, of course, model dependent, but the underlying argument should be of general validity.

In a similarly crude way the effect of secondary production upon lifetimes may be calculated. Substitution of  $\gamma_{TOT}(D')$  for  $\gamma(D')$  in equation 11 leads to a shortening of lifetimes, and for a given time a consequent shift of the limit of equilibrium to larger diameter. The equilibrium distribution itself, being independent of the amplitude of the production mechanism, will be largely unaffected by the incorporation of secondary cratering.

#### INTERPRETATION OF EXPERIMENTAL COUNTS

The models of cratering sketched here are crude but should contain the essential features of the process, and an attempt can be made to recognize in the experimental counts features reflecting the mechanisms. It should be possible to discriminate between three diameter ranges:

- (i) large craters for which the distribution is determined by the meteoroid flux and secondary cratering is unimportant,

- (ii) small craters for which the distribution is in equilibrium and, at least in one model, determined principally by geometrical factors, and to a subsidiary degree by local surface properties and
- (iii) an intermediate region not in equilibrium but where secondary cratering is important.

Estimates of the position of the boundaries depend upon the details of the models used, but to an order of magnitude secondary cratering should become important at  $\sim 100$  meters diameter. For a surface  $10^9$  years old (a typical age quoted for regions of the moon) the limiting size for equilibrium is  $10 \sim 100$  meters [Collins 1965]. Whether or not these boundaries are discernable in the experimental counts is a matter of opinion.

The major division in the counts is between highlands and maria and is probably too great to be attributable solely to the difference in properties of two equilibrium surfaces. The alternative conventionally accepted is that the maria have not yet reached equilibrium for the range of diameters observed and are younger than the highlands which have apparently reached equilibrium [e.g., Young, 1933].

Lesser differences in amplitude within the two main distribution groups may reflect differences in age (in the maria) or differences in material properties (in maria and highlands). The ambiguity exhibited here is serious and cannot be resolved by a study of crater statistics alone, but in individual instances it may be illuminated by a qualitative investigation of the region, exemplified by the study of Ptolemaeus and its environs by Palm and Strom [1963]. Residual differences between saturation distributions can probably be attributed to local variation in surface material properties and may prove a useful indicator in this connection.

A further piece of experimental evidence is available to fit into an impact theory. Some of the ejecta from impact craters with insufficient energy to produce secondaries will be distributed over the surface as visible fragments. Collins [1966] has developed a theoretical model in which the production of fragments competes with the destructive mechanisms of burial and micrometeoroid erosion. The fragment size density distribution predicted by the theory compares well with distributions measured in the Luna IX panoramas [Smith, 1967] and in photographs taken by Surveyor I [Rennilson et al. 1966]. Differences in amplitude may again be interpreted in terms of surface age or consistency.

Comparison of the predictions of different theoretical models with experiment [Brinkmann, 1966; Collins, 1965, 1966; Marcus, 1966a, b; Shoemaker, 1965] is in each case reasonably satisfactory but not sufficiently close for the exclusive justification of any one model. The situation could be improved in two ways. Detailed crater counts extending over several orders of diameter magnitude penetrating to meter size craters or further (of the type published by Shoemaker [1966], from an analysis of Ranger photographs) and the investigation of equilibrium and non-equilibrium surface cratering in the laboratory would advance the experimental position. Theoretically, more realistic models of the primary and secondary processes are required to fit into a general statistical framework for cratering, for example that presented by Marcus [1964].

With these improvements it seems likely that the study of crater and fragment distribution statistics in conjunction with qualitative judgment may provide detailed information about both age and material properties of the lunar surface.

*RJ Collins (BGS)*

R. J. Collins

*BG Smith*

B. G. Smith

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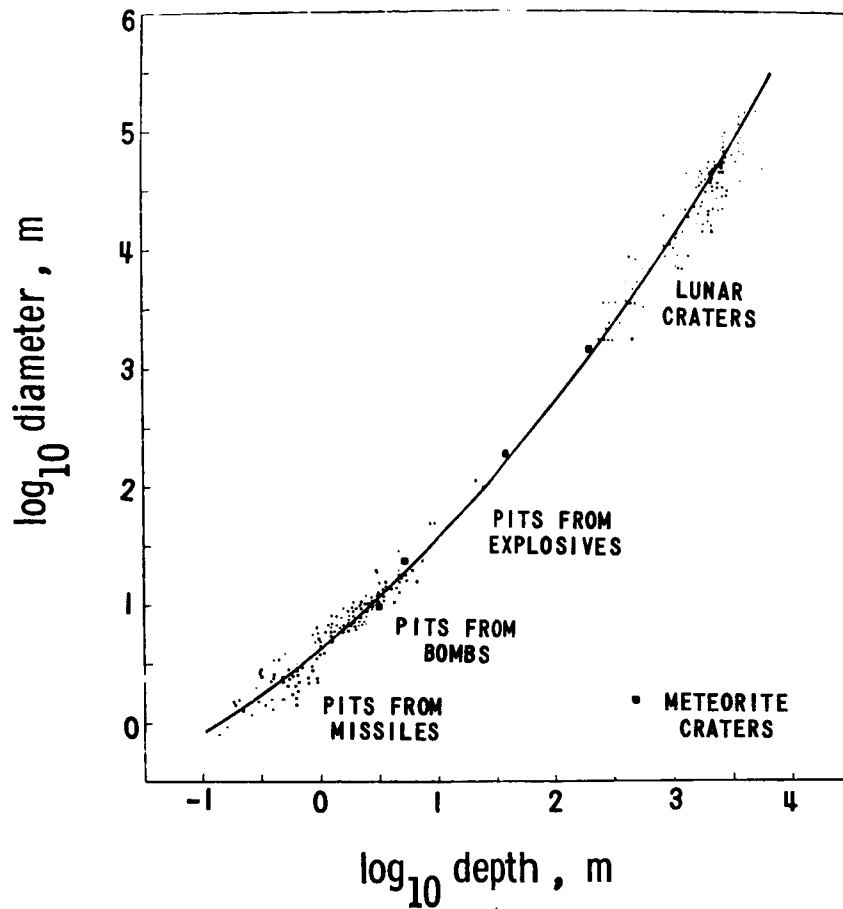


Fig. 1 : Diameter-depth relation for various types of craters (after Baldwin 1949)

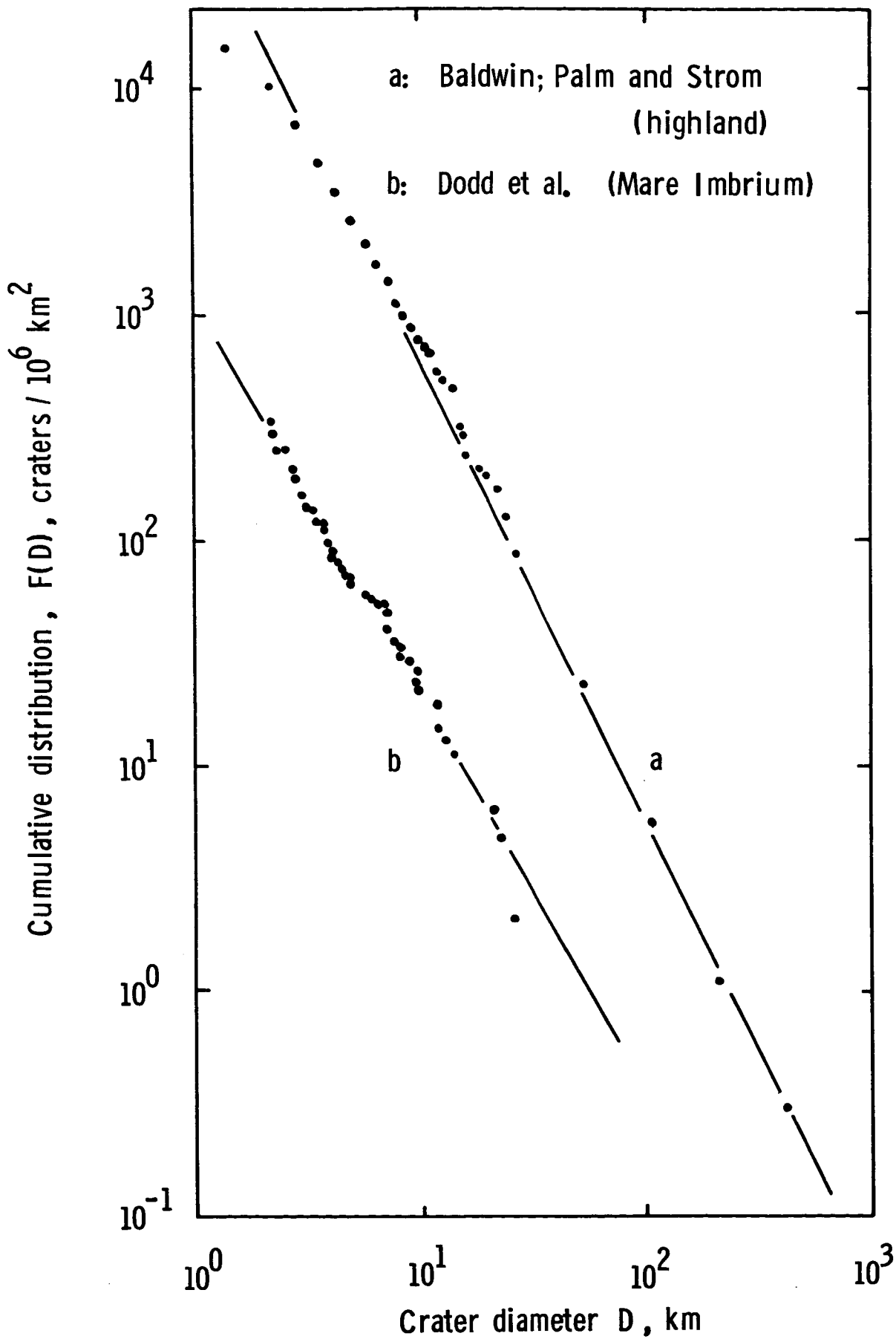


Fig. 2 : Cumulative crater distributions, after Baldwin (1964) and Dodd, Salisbury and Smalley (1963)

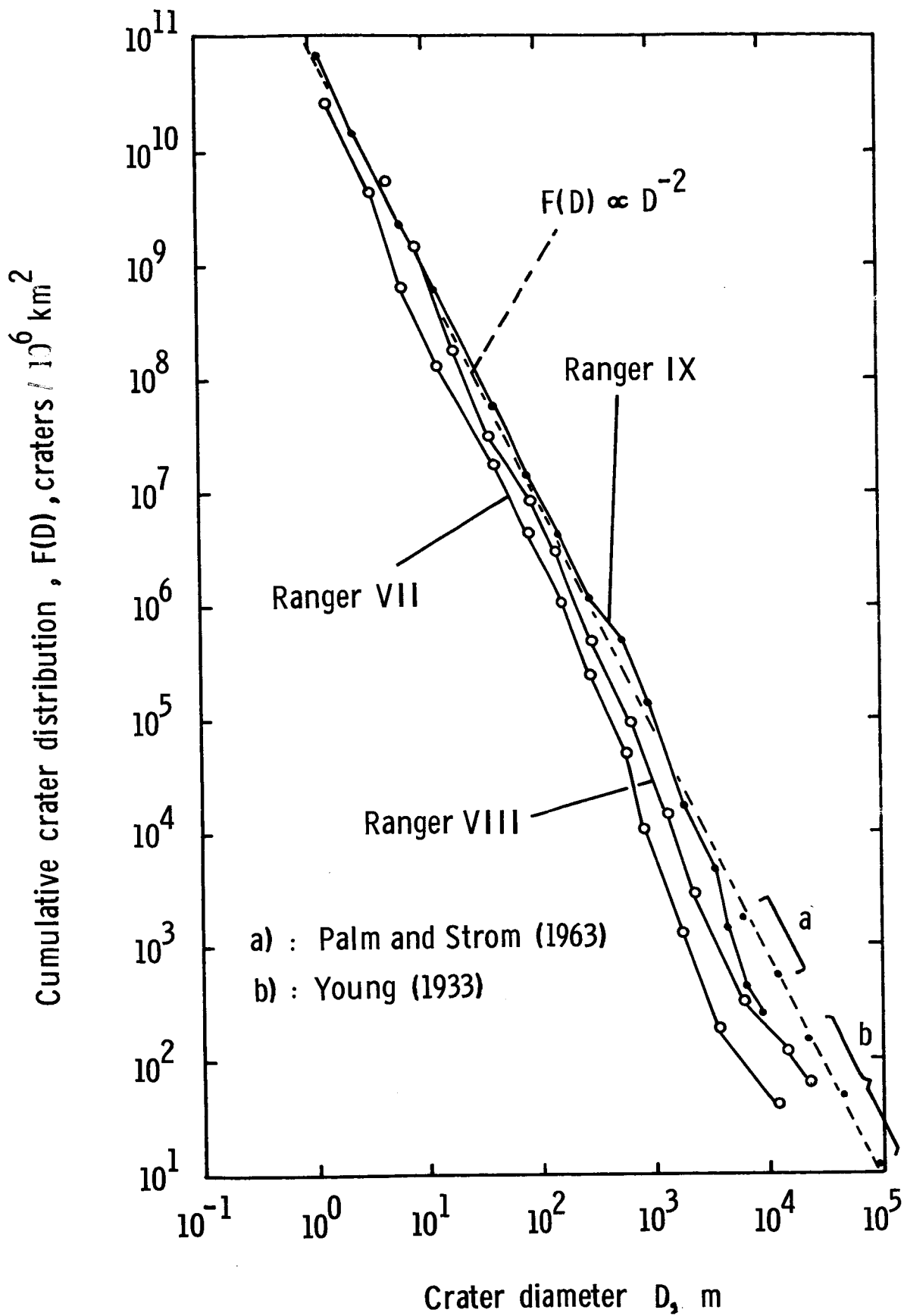


Fig. 3 : Cumulative crater distributions, after Shoemaker (1966)

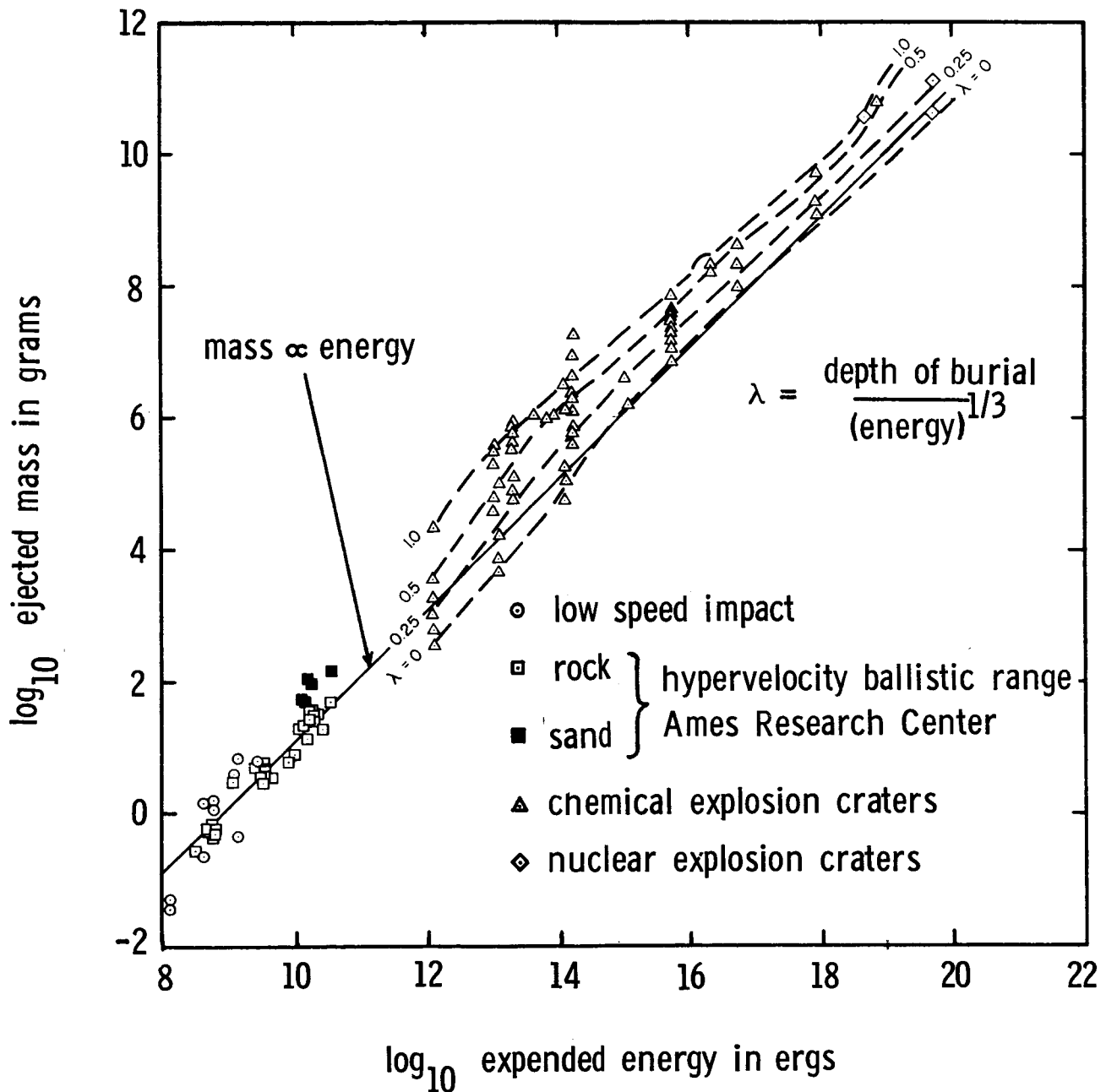


Fig. 4 , Correlation of the total ejected mass with expended energy for explosive and impact cratering events (after Gault et al. 1963)

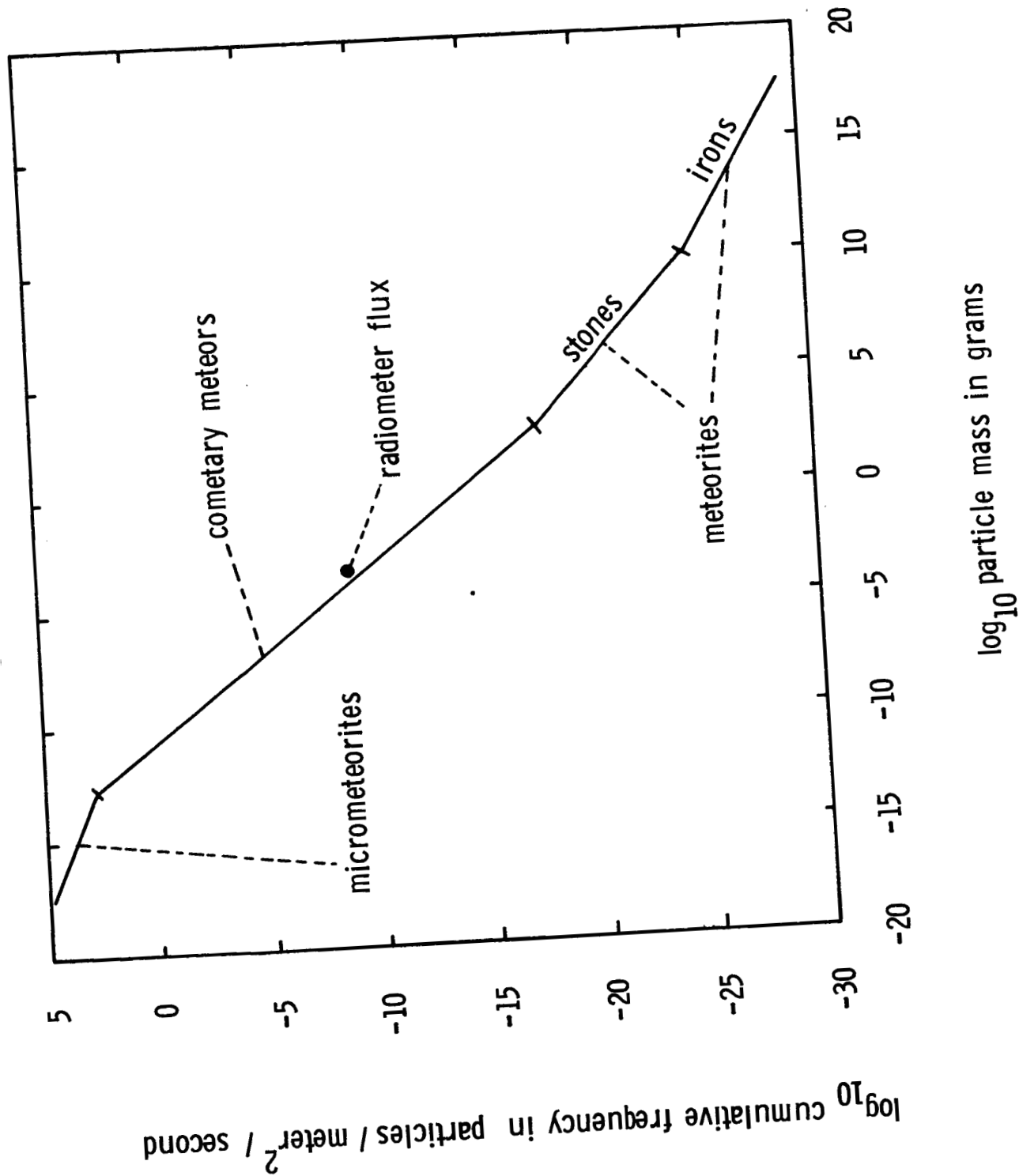


Fig. 5: Cumulative frequency-mass distribution for interplanetary debris quoted by Hawkins (1964)

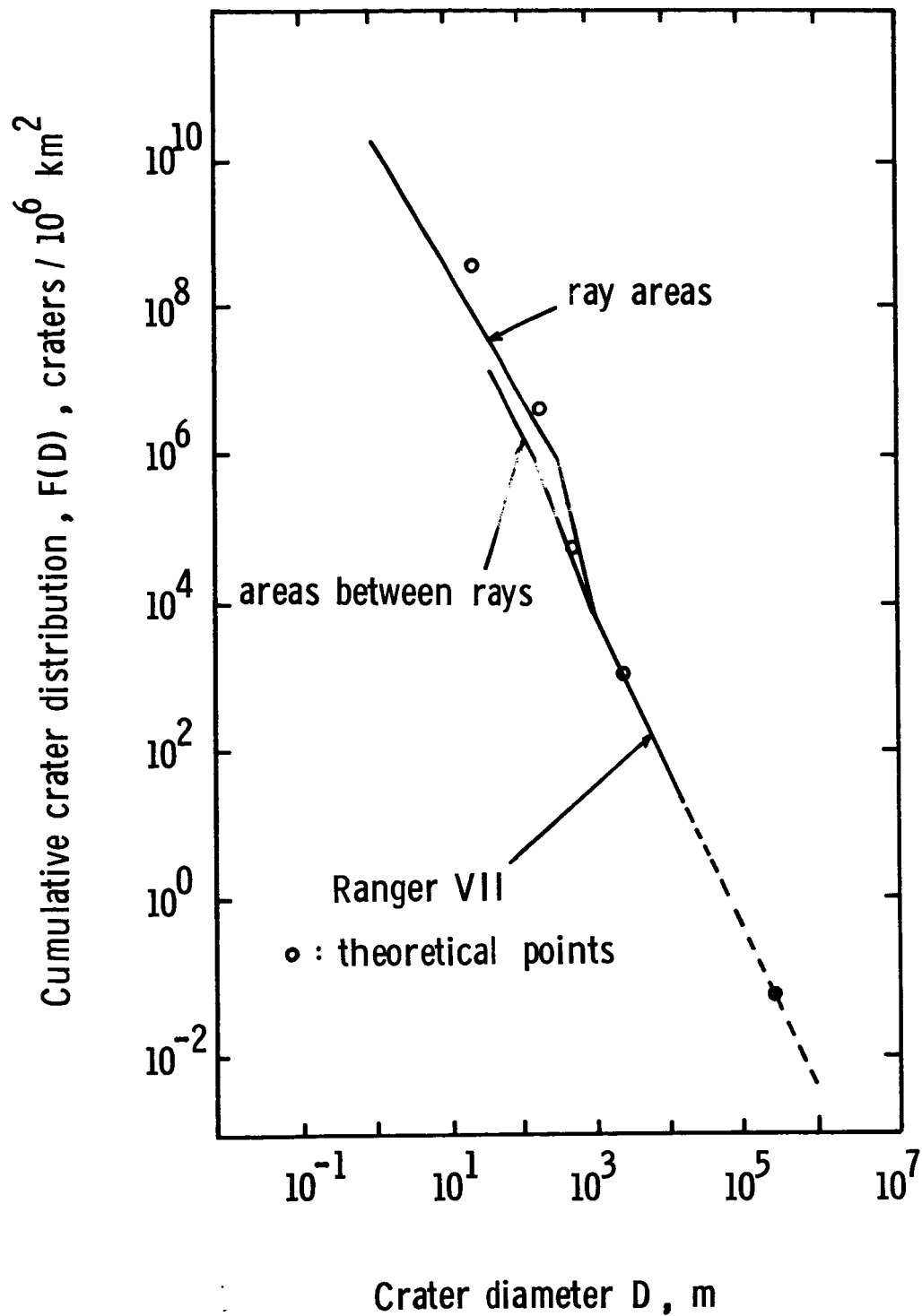


Fig. 6 : Crater counts from Ranger VII ( Shoemaker 1965 )  
 compared with Collins' theoretical model



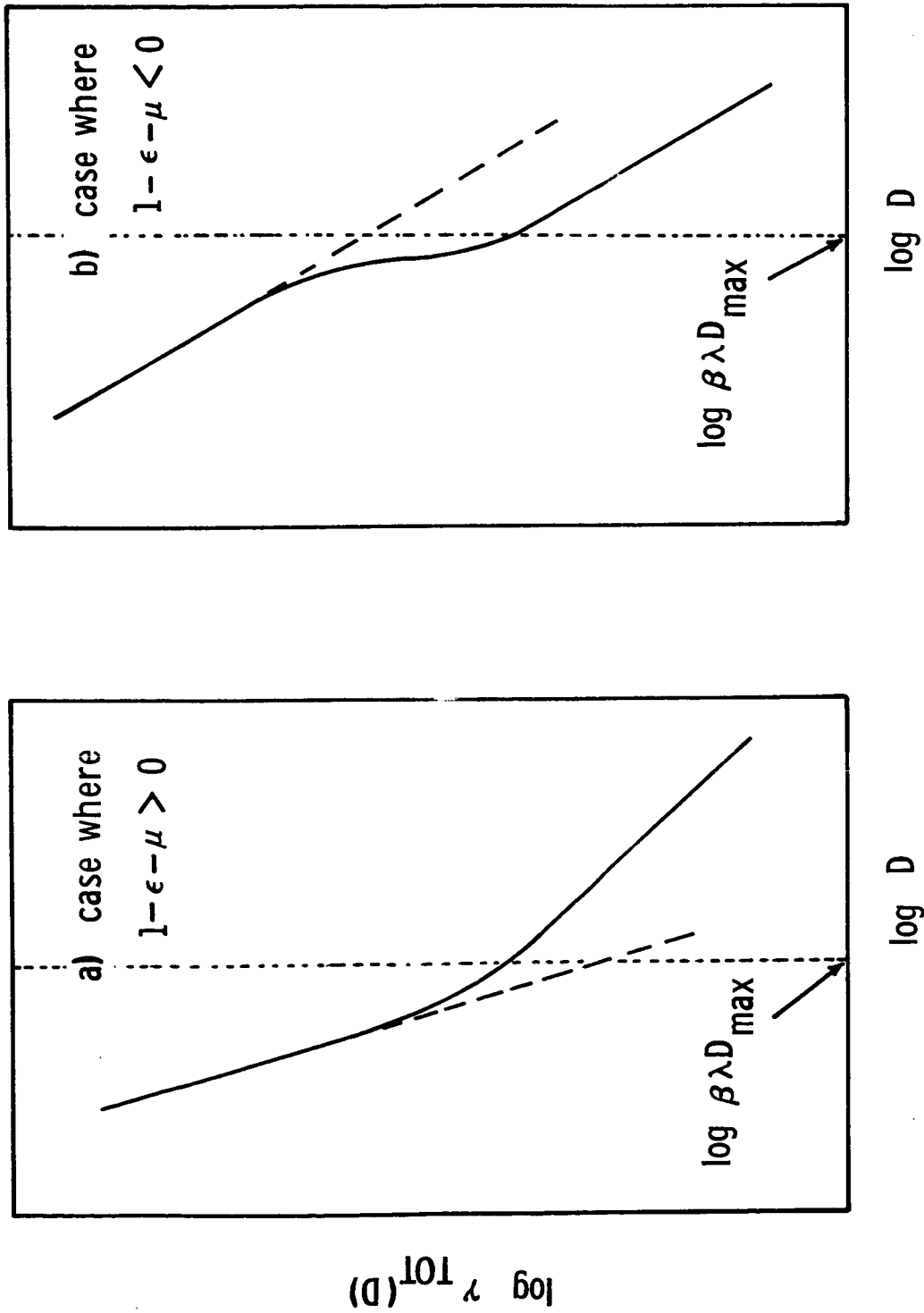


Fig. 7 : Diagrammatic interpretation of  $\gamma_{TOT}(D)$ , equation 20, illustrating the importance of secondary cratering

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