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TECHNICAL MEMORANDUM

DENSITY AND FLUX DISTRIBUTIONS IN THE LUNAR ATMOSPHERE DUE TO POINT AND LINE SOURCES

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ABSTRACT

Apart from the globally distributed ambient gases in the lunar atmosphere, localized gas sources, such as volcanos, may have a measurable effect on the process of shaping the lunar atmosphere. Analytic solutions have been obtained for the spatial distributions of neutral gas densities and fluxes due to point and line sources at the lunar surface. Both density and flux profiles are strong functions of the distance from the source to the point of observation. For a scale height of 25 km the density at an altitude of 110 km decreases by four orders of magnitude as the horizontal distance is increased from 0 to 500 km from the source. The study also reveals: (1) the location of the source may be identified from the density and flux profiles; (2) if the gas species is known, the temperature of the gas may be determined from the density gradient at a distance of several scale heights from the source, and (3) the strength of the gas source may be determined by the magnitudes of density and flux. To distinguish between point and line sources, careful data analysis is necessary. In addition, it is best to measure simultaneously density as well as fluxes from two orthogonal directions.

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TECHNICAL MEMORANDUM

Introduction

One of the primary scientific objectives of the Apollo missions is to understand the dynamic processes shaping the lunar atmosphere. To achieve this objective, various experiments have been designed to measure the composition and distribution of the lunar atmosphere. The Orbital and Surface Mass Spectrometer Experiments of the Apollo 15, 16 and 17 missions and the Cold Cathode Gauge Experiments of the Apollo 14 and 15 missions are for this purpose. To support these experiments, a study, based on the kinetic theory of gases, was recently made by Yeh and Chang (1971), in which they calculated numerically the density and flux distributions of neutral gases in the lunar atmosphere resulting from assumed gas temperature and density variations over the entire surface. In that investigation, emphasis was placed on the global distribution of the ambient atmosphere as the result of gas transport due to large scale variations of gas temperatures and concentrations at the surface. However, localized gas sources should have additional effects on the ambient atmosphere. These gas sources could be volcanic point sources or line sources related to release of gases along fault planes or fracture zones, or along the terminator. In fact, these outgassing events are of great interest because they may be related to other dynamic processes in the lunar interior.

Consequently, in the present investigation, by using the same approach of Yeh and Chang (1971), we calculate the density and flux distributions in the atmosphere due to point and line sources of gas at the surface. Comparison of the results derived here with those of the Mass Spectrometer Experiments should enable identification of the location and determination of the strength as well as of the temperature of the gas sources.

Gas Transport Model

For the study of the neutral gas transport in the lunar atmosphere, an exospheric model has been used by many investigators (e.g., Hodges and Johnson, 1968; Vogel, 1966; Yeh and Chang, 1971). Here we follow that of Yeh and Chang (1971). In this model, the base of the exosphere is the surface of the moon. All the gas particles are originating from (or underneath) the surface and travel with free trajectories through the exosphere under the influence of the lunar gravitational force until they land on the surface or escape to space. In other words, collisions between gas particles and non-thermal escape mechanisms such as photo-ionization and subsequent removal by the solar wind are neglected.

In this model, if a particle originates at a point on the surface, where the coordinate is $\underline{x}_o$ and its velocity vector is $\underline{v}_o$, its position $\underline{x}$ and velocity $\underline{v}$ at a later time can be calculated from the trajectory equations. Conversely, if an observer located at a position $\underline{x}$ in space sees a particle having a velocity vector $\underline{v}$, he can, by following the particle's trajectory, always trace back its initial position $\underline{x}_o$ and velocity $\underline{v}_o$ at the surface. By summing over all possible velocities, an observer at $\underline{x}$ counts all the particles originating from the lunar surface that reach $\underline{x}$. If a statistical analysis with appropriate weighting is performed, the density and fluxes can be easily obtained.

For localized gas sources, considerable simplification in calculation is achieved, if we exclude light gases such as hydrogen and helium. For gases heavier than helium, the lifetime of particles due to thermal escape is long compared with the transport time. The average distance traveled by these particles is also small compared with the lunar radius. Consequently, a flat surface with constant gravitational field becomes a good approximation. With these assumptions, analytic solutions of density and flux distributions are obtained for localized point and line sources.

Kinetic Equation

The neutral particles (molecules, atoms) may be described by the velocity distribution function $f(\underline{x}, \underline{v}, \underline{t})$ which, in the absence of collisions, obeys the collisionless

Boltzmann equation,

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f + \underline{F} \cdot \nabla_v f = 0 \quad , \tag{1}$$

where $\underline{F}$ is the force per unit mass acting on a particle, and ∇ and ∇_v are respectively the gradient operators in position and velocity space. Once the distribution function is solved from Eq. (1), all the macroscopic quantities can be obtained by taking moments of f. In particular, the density is

$$n(\underline{x}, t) = \int f(\underline{x}, \underline{v}, t) \, d^3v \quad , \tag{2}$$

and the particle flux in the i-direction is

$$\underline{\psi}_i (\underline{x}, t) = \int \underline{v}_i \, f(\underline{x}, \underline{v}, t) \, d^3v \quad , \tag{3}$$

where $\underline{v}_i$ is the velocity along the direction i. In general, there will be a distribution function for each species of gas. In such a case, a subscript j may be attached to f to indicate that particular species.

The general solution of Eq. (1), which is well known, is that f is any arbitrary function of the integrals of the Lagrangian subsidiary equations,

$$\frac{d\underline{x}}{dt} = \underline{v} \quad , \qquad \frac{d\underline{v}}{dt} = \underline{F} \quad . \tag{4}$$

Eqs. (4) are just the equations of motion of the particle under the force field $\underline{F}$. These equations are also referred to as the trajectory equations in phase space. In fact, the statement for the general solution of f means that f is a constant along a trajectory in phase space. In other words, if f is known at one time, it is conserved for all times along a trajectory in phase space. The particular value of f at ($\underline{x}$, $\underline{v}$, t) is, therefore, determined by the particular trajectory in question and the initial distribution functions at the boundary $f_o(\underline{x}_o, \underline{v}_o, o)$. Consequently, $f(\underline{x}, \underline{v}, t)$ may be expressed by the following relation

$$f(\underline{x}, \underline{v}, t) = \int f_o(\underline{x}_o, \underline{v}_o, o)\ \delta(\underline{x}-\underline{X})\ \delta(\underline{v}-\underline{V})\ d^3x_o\ d^3v_o \ , \qquad (5)$$

where

$$\underline{x} = \underline{X}(\underline{x}_o, \underline{v}_o, t) \ ,$$

and (6)

$$\underline{v} = \underline{V}(\underline{x}_o, \underline{v}_o, t) \ ,$$

are the position and velocity trajectories of a single particle parameterized in terms of its initial conditions $\underline{x}_o$ and $\underline{v}_o$ as well as time. The present investigation will be restricted to the steady state case. This means that the results are valid only if the duration of the gas emission is long compared with the time scale of interest, such as the time it takes a spacecraft to pass over the gas source. In the steady state case, the time is considered as a free parameter in Eq. (6).

Particle Trajectories

For a flat surface model, it is convenient to choose Cartesian coordinates centered at the source. The trajectory equations in this coordinate system are simply the following:

$$x = x_o + u_o t \ , \quad y = y_o + v_o t \ ,$$

$$z = z_o + w_o t - \frac{1}{2} g t^2 \ ; \tag{7}$$

$$u = u_o \ , \quad v = v_o \ , \quad w = w_o - gt \ ;$$

where g is the lunar gravitational field, and u, v and w are the velocity components in x, y and z directions. In a steady state case, the time t is given by

$$t = \left(\sqrt{w^2 + 2g(z - z_o)} - w \right) / g \tag{8}$$

Density and Flux Distributions of a Point Source

In order to determine the density of particles at a particular location $\underline{x}$ in space, it is only necessary to integrate the distribution function f with respect to velocities, namely

$$n = \int f \, d^3v$$
$$= \int f_o \, \delta(\underline{x}-\underline{X}) \, \delta(\underline{v}-\underline{V}) \, d^3x_o \, d^3v_o \, d^3v \tag{9}$$

For simplicity, we shall assume that f is Maxwellian at the surface, characterized by a temperature T which may or may not be the same as the surface temperature of the moon. Furthermore, the source is assumed to be a delta function located at the origin. The strength of the source, defined as the total number of particles per unit time emerging from the source, is expressed by $\dot{N}$. Under these assumptions, the boundary condition of f at the surface may be written as

$$f_o = \int dt \, \dot{N} \, \delta(x_o) \, \delta(y_o) \, \delta(z_o) [m/2\pi kT]^{3/2} \exp[-m\underline{v}_o{}^2/2kT] \ , \tag{10}$$

where

$$\underline{v}_o{}^2 = u_o^2 + v_o^2 + w_o^2 \ . \tag{11}$$

Substituting (7), (8), (10) and (11) into (9), the resulting density is*

$$n = \frac{\dot{N}}{2\pi^2 \, H \, \langle v \rangle} \, \frac{1}{r} \exp(-Z/2H) \, K_1(r/2H) \ , \tag{12}$$

* The result of Eq. (12) was also obtained independently by R. R. Hodges, Jr.

where,

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \quad , \tag{13}$$

$$H = kT/mg \quad , \tag{14}$$

$$\langle v \rangle = (8\ kT/\pi m)^{\frac{1}{2}} \quad , \tag{15}$$

and K_1 is the modified Bessel function of the second kind of order one. The quantity H has a dimension of length and is the same as the scale height defined for a homogeneous atmosphere; and $\langle v \rangle$ has a dimension of velocity and is the same as the averaged speed of the gas particles in thermal equilibrium.

Similarly, the particle fluxes in x and y directions are, by definition,

$$\underline{\psi}_x = \underline{i} \int u\, f\, d^3v \quad ,$$

$$\underline{\psi}_y = \underline{j} \int v\, f\, d^3v \quad ,$$

and the results for the boundary condition (10) are

$$\underline{\psi}_x = \underline{i}\, \frac{\dot{N}}{4\pi}\, \frac{x}{r^3}\, (1 + \frac{r}{2H})\, \exp[-(r+z)/2H] \quad , \tag{16}$$

$$\underline{\psi}_y = \underline{j}\, \frac{\dot{N}}{4\pi}\, \frac{y}{r^3}\, (1 + \frac{r}{2H})\, \exp[-(r+z)/2H] \quad , \tag{17}$$

where $\underline{i}$ and $\underline{j}$ are unit vectors in x and y directions.

Density and Flux Distributions of a Line Source

For a line source located on the y axis, we assume that f_o has the following form,

$$f_o = \int dt\ \dot{M}\ \delta(x_o)\ \delta(z_o)\,[m/2\pi kT]^{3/2}\ \exp[-m\underline{v}_o^{\,2}/2kT]\ , \qquad (18)$$

where $\dot{M}$ is the number of particles per unit time per unit length emerging from the line source. For this distribution function, the density and x and y fluxes are the following

$$n = \frac{\dot{M}}{\pi\langle V\rangle}\,\frac{1}{\ell}\,\exp[-(\ell+z)/2H]\ , \qquad (19)$$

$$\underline{\psi}_x = \underline{i}\,\frac{\dot{M}}{4\pi H}\,\frac{x}{\ell}\,\exp(-z/2H)\ k_1(\ell/2H)\ , \qquad (20)$$

$$\underline{\psi}_y = \underline{j}\,\frac{\dot{M}}{\pi}\,\frac{1}{\ell}\,\exp[-(\ell+z)/2H]\ , \qquad (21)$$

where,

$$\ell = (x^2 + z^2)^{1/2}\ . \qquad (22)$$

Discussion of Results

It is convenient to normalize the results obtained in the previous two sections. If we define for a point source

$$n_p = \dot{N}/(\pi^2 H^2 \langle v \rangle) \quad , \tag{23}$$

$$\psi_p = \dot{N}/(4\pi H^2) \quad , \tag{24}$$

then the non-dimensional density and fluxes (magnitudes) become

$$\frac{n}{n_p} = \frac{H}{2r} \exp(-z/2H) \; K_1(r/2H) \quad , \tag{25}$$

$$\frac{\psi_x}{\psi_p} = \left(\frac{H}{r}\right)^2 \frac{x}{r} \left(1 + \frac{r}{2H}\right) \exp[-(r+z)/2H] \quad , \tag{26}$$

$$\frac{\psi_y}{\psi_p} = \left(\frac{H}{r}\right)^2 \frac{y}{r} \left(1 + \frac{r}{2H}\right) \exp[-(r+z)/2H] \quad . \tag{27}$$

The reason for the choice of (23) and (24) as the normalization factors is that the ratio of ψ_p to n_p, which may be defined as a reference average velocity of the gas particles, is, except for the directional factor x/r, equal to the ratio of ψ_x to n [Eqs. (20) and (19)] in the limit of $(r/H) \to 0$, i.e., near the source. The normalization constants (23) and (24) depend only on the source strength $\dot{N}$ and the temperature of the particular gas particles in question.

Similarly, for a line source, we define

$$n_L = \frac{\dot{M}}{\pi H \ \langle v \rangle} \quad , \tag{28}$$

$$\psi_L = \frac{\dot{M}}{2\pi H} \quad , \tag{29}$$

and the normalized density and fluxes are

$$\frac{n}{n_L} = \frac{\psi_y}{2\psi_L} = \frac{H}{\ell} \exp[-(\ell+z)/2H] \quad , \tag{30}$$

$$\frac{\psi_x}{\psi_L} = \frac{x}{2\ell} \exp(-z/2H) \ K_1(\ell/2H) \quad . \tag{31}$$

The normalized density and flux profiles of point and line sources are plotted in Figs. 1-5 as functions of x for several values of y and at z=110 km (≈60 nm). Two groups of curves are shown, one for H=25 km, and another for H=50 km. The value of H=25 km, for instance, corresponds to a temperature of ∼100°K for neon and ∼200°K for argon. The density profiles are strong functions of the distance from the sources, both for point and line sources. For example, in Fig. 1, for the case of H=25 km, the density decreases by four orders of magnitude as x is increased from 0 to 500 km. The gradients of both density and fluxes (on a logarithmic scale) with respect to x tend to be approximately constant for $x > 200$ km; however, the values of these constants differ for different cases, as illustrated by the distinct slopes of the solid curves in Fig. 1 for the cases of H=25 km and H=50 km. Therefore, for a known gas species, the density gradient may be used to determine the temperature of the gas source, since $H=kT/mg$. This is similar to the case of the earth atmosphere at high altitude where the temperature is determined by the vertical density gradient.

Comparison of Fig. 1 with Fig. 4, or of Fig. 2 with Fig. 5, reveals the relative insensitivity of density and flux distributions in space to the nature of the source; the shape of the respective curves appears similar for both point and line sources. Obviously, the asymmetry of density and flux gradients in the orthogonal directions should in general bring out the distinction between the two types of sources. But even under circumstances when this is not possible, examination of the density and flux distribution at large distances from the source can demonstrate the distinction.

The asymptotic behavior of K_1 for large argument is

$$K_1(\zeta) \sim \sqrt{\frac{\pi}{2\zeta}}\; e^{-\zeta}\,[1 + O(\zeta^{-1})] \quad . \tag{32}$$

The above asymptotic approximation differs by only a few percent from its exact value when the argument ζ is greater, say, than 5, which corresponds to 250 km for r in the case of H=25 km.

This distance is much smaller than the radius of the moon and the present flat surface model is consistent with the findings. Thus, the normalized density distribution at large distances from a point source may be written as

$$\frac{n}{n_p} \sim \frac{\sqrt{\pi}}{2} \left(\frac{H}{r}\right)^{3/2} \exp[-(r+z)/2H] \; [1 + 0(\frac{H}{r})] \quad . \qquad (33)$$

Comparing Eq. (33) with the result of line source, which is

$$\frac{n}{n_L} = \frac{H}{\ell} \exp[-(\ell+z)/2H] \quad , \qquad (30)$$

we see that the density due to a point differs from that of a line source by a factor inversely proportional to the square root of distance from the source. A similar condition holds for the x-flux. However, because both Eq. (30) and Eq. (33) are dominated by the exponential factor, careful data analysis is needed in determining the nature of the source. To achieve this purpose, it is preferable to perform simultaneous measurements of density as well as of fluxes in orthogonal directions.

A contour map of equal-density curves at an altitude of z=110 km resulting from a point and a line source are shown together in Fig. 6. The results are normalized at zero horizontal distances from the sources. The concentric circles are constant density curves due to a point source, and the parallel straight lines are those due to a line source in an arbitrary direction with respect to the spacecraft trajectory. In the figure, the ζ and η axes are chosen to be in and perpendicular to the direction of the spacecraft's trajectory. In this particular case, the ζ-axis is 100 km from the point source, and intersects the line source with an angle of 60°. This illustrates the density variations along a spacecraft's trajectory in the given situation.

When the spacecraft's trajectory is not perpendicular to the line source, both fluxes in ζ and η directions as functions of ζ are asymmetric with respect to $\zeta=0$. This is shown in Fig. 7 for the case of the line source at 30° with the η axis. For instance, the ratio of ψ_ξ at $\xi = -300$ km to that at $\xi = 300$ km is ~ 0.27; and the corresponding value for ψ_η is ~ 1.85. Therefore, such asymmetry properties may give information on the orientation of the line source.

Summary and Conclusions

On the basis of the kinetic theory of gases, analytic solutions have been obtained for the spatial distributions of neutral gas densities and fluxes due to point and line sources at the lunar surface. Particular examples of density and flux variations with distance along the ground track at an altitude of 110 km were shown. Three major findings may be summarized as follows: (1) the location of the source may be identified from the density and flux profiles, (2) the temperature of the gas source may be determined from the asymptotic behaviors of density and flux gradients at large x, if the gas species is known (e.g., by mass spectrometer), and (3) the strength of the gas source may be determined by the magnitudes of the measured density and fluxes.

To distinguish the type of the gas source, i.e., whether it is a point or a line source, extremely careful data analysis is necessary, since the difference between measurements in orbit for these two types of sources may be small. To achieve the above purpose, it is best to perform simultaneous measurements of density as well as x, y-fluxes, which may require a change in spacecraft attitude from one orbit to the next.

A final remark is due on the limitations of the results obtained in the present investigation. These results are valid for gases whose molecular weight are not too small; hydrogen and helium are excluded. However, since the Apollo Mass Spectrometer experiments are designed to measure gases with mass number ranging from 12 to 66, the present analysis is quite adequate. In addition, the results are valid only at the night side of the moon where the surface temperature is low and the gas particles impinging on the surface are absorbed. The results are also restricted to gas sources that persist long enough compared with the time scale of interest, such as that of the spacecraft passing by the gas source. Instantaneous venting of gas (in the above sense) requires treatment by time dependent analysis.

Acknowledgment

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T. T. J. Yeh

for G. K. Chang

2015-$\frac{\text{TTJY}}{\text{GKC}}$-ams

Attachments

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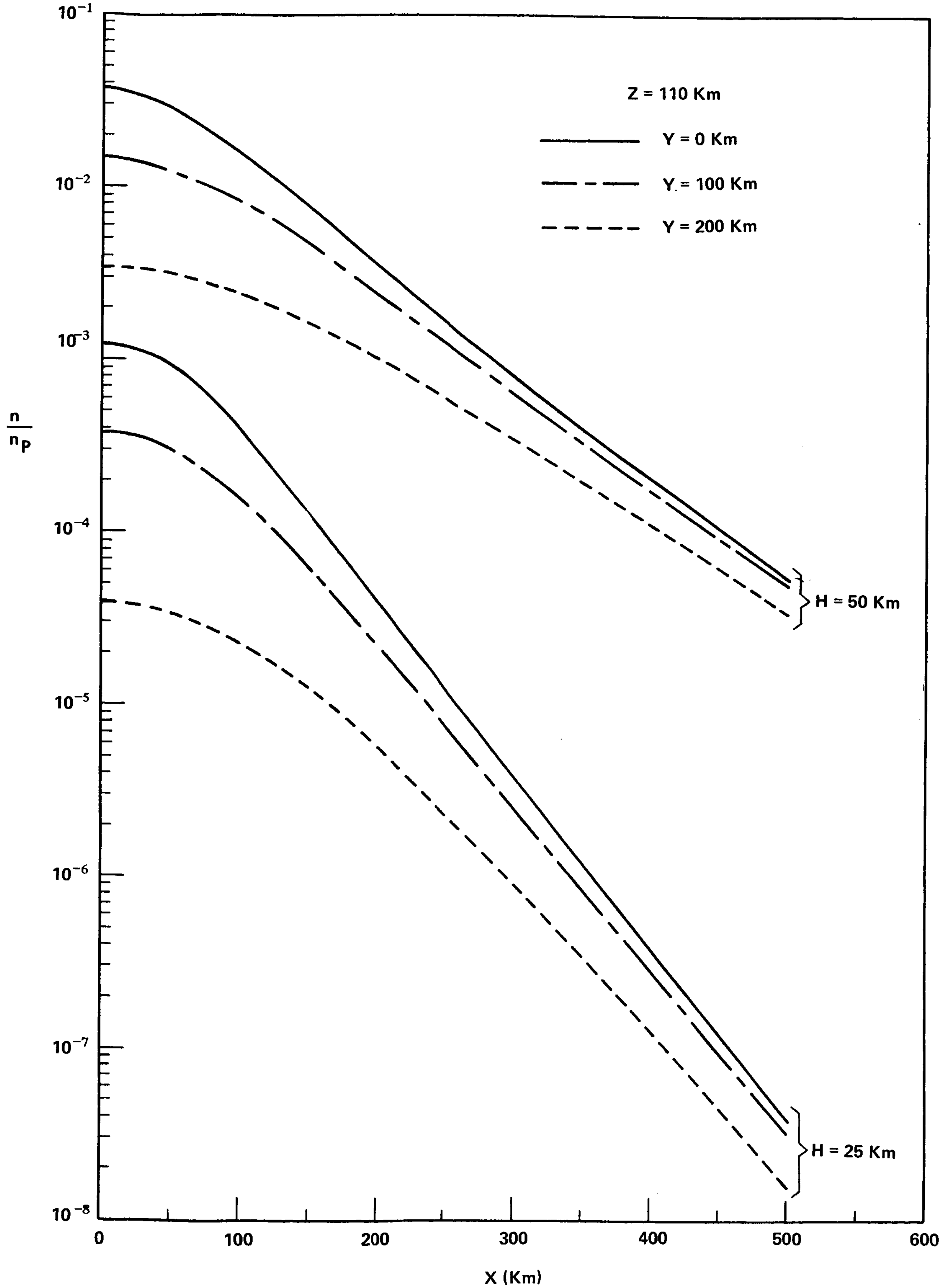

FIGURE 1 - DENSITY VARIATION VS X AT Z = 110 Km IN THE CASE OF A POINT SOURCE

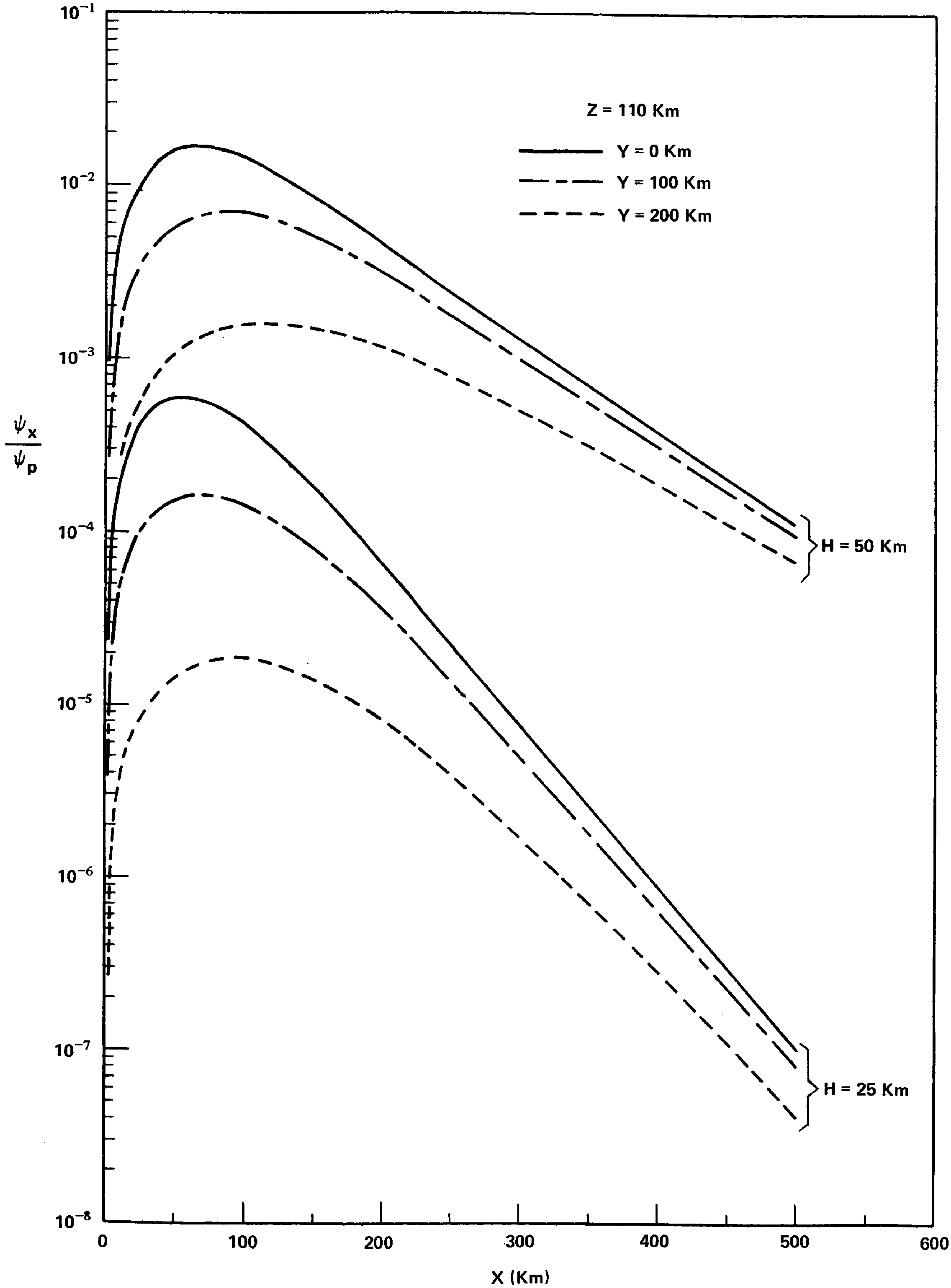

FIGURE 2 - X-FLUX VARIATIONS VS X AT Z = 110 Km IN THE CASE OF A POINT SOURCE

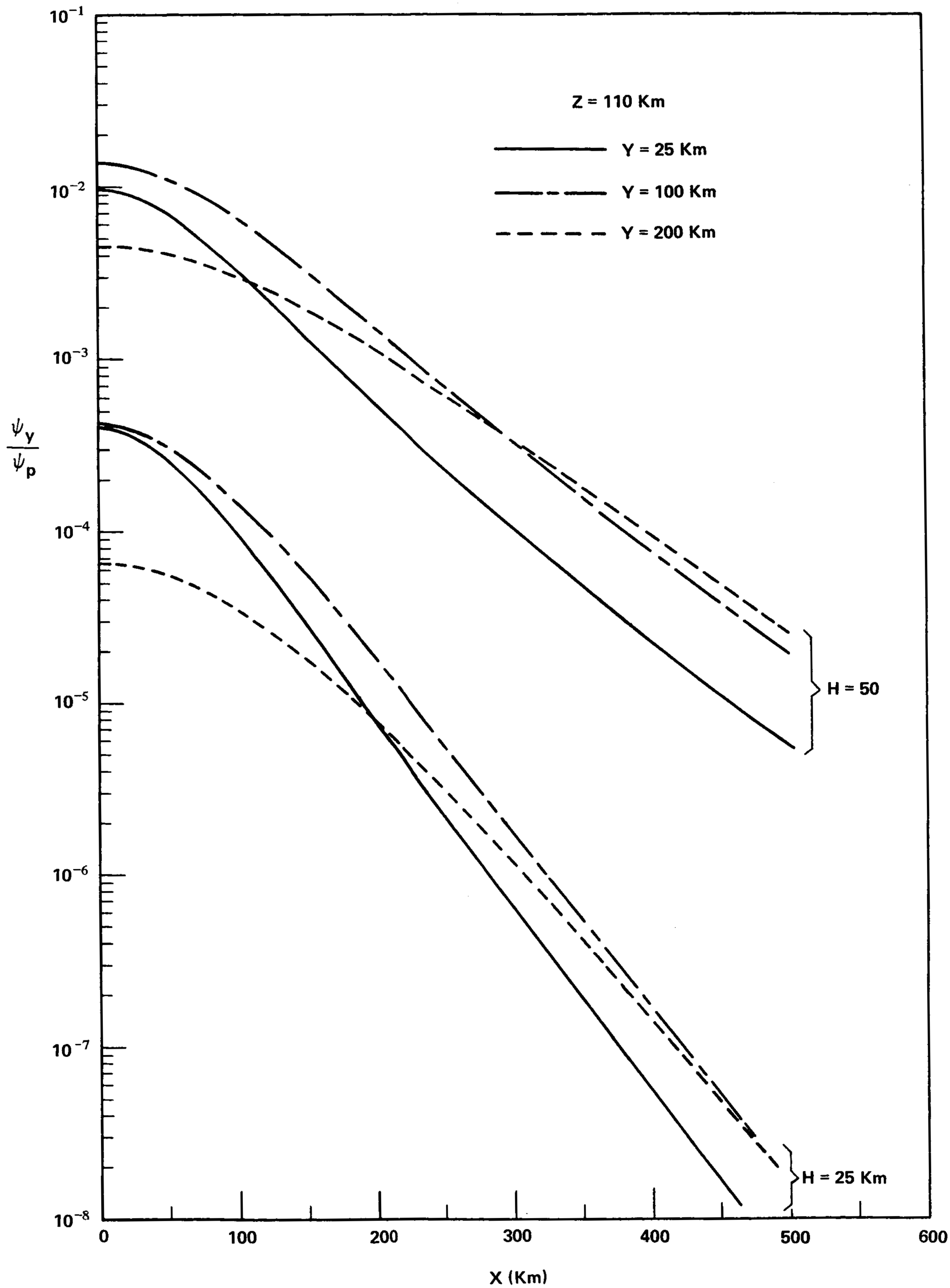

FIGURE 3 - Y-FLUX VARIATIONS VS X AT Z = 110 Km IN THE CASE OF A POINT SOURCE

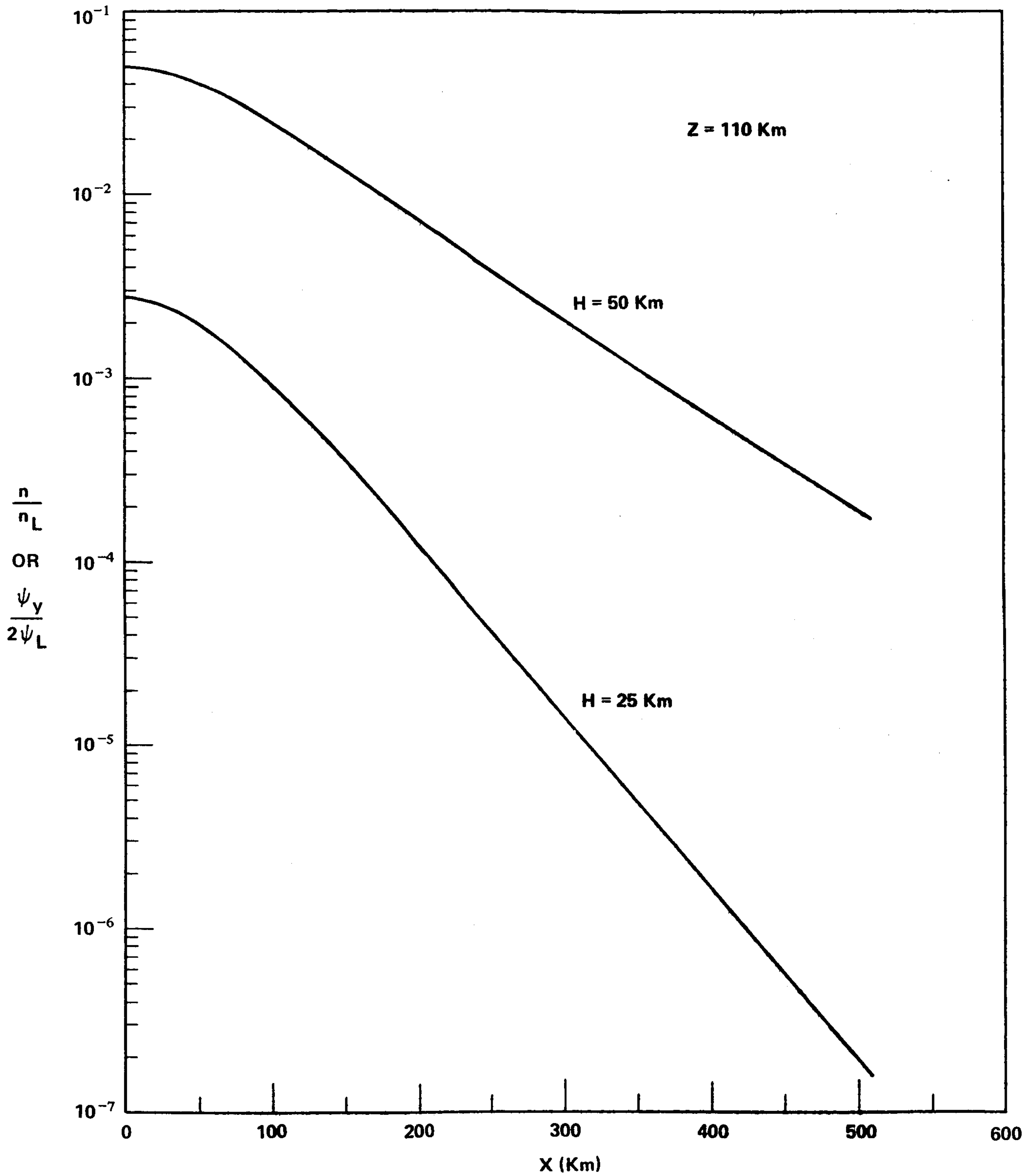

FIGURE 4 - DENSITY AND Y-FLUX VARIATIONS VS X AT Z = 110 Km IN THE CASE OF A LINE SOURCE

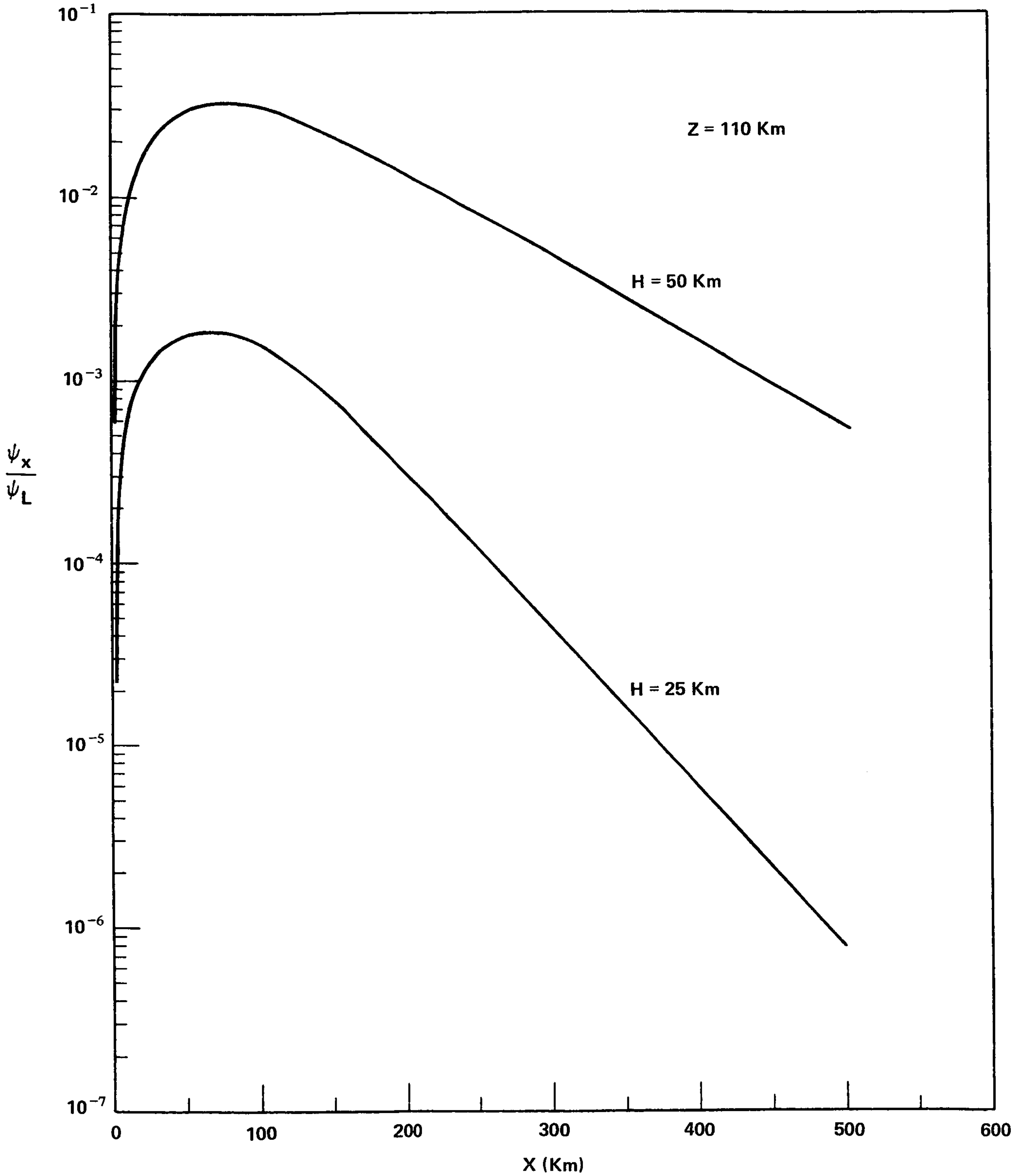

FIGURE 5 - X-FLUX VARIATIONS VS X AT Z = 110 Km IN THE CASE OF A LINE SOURCE

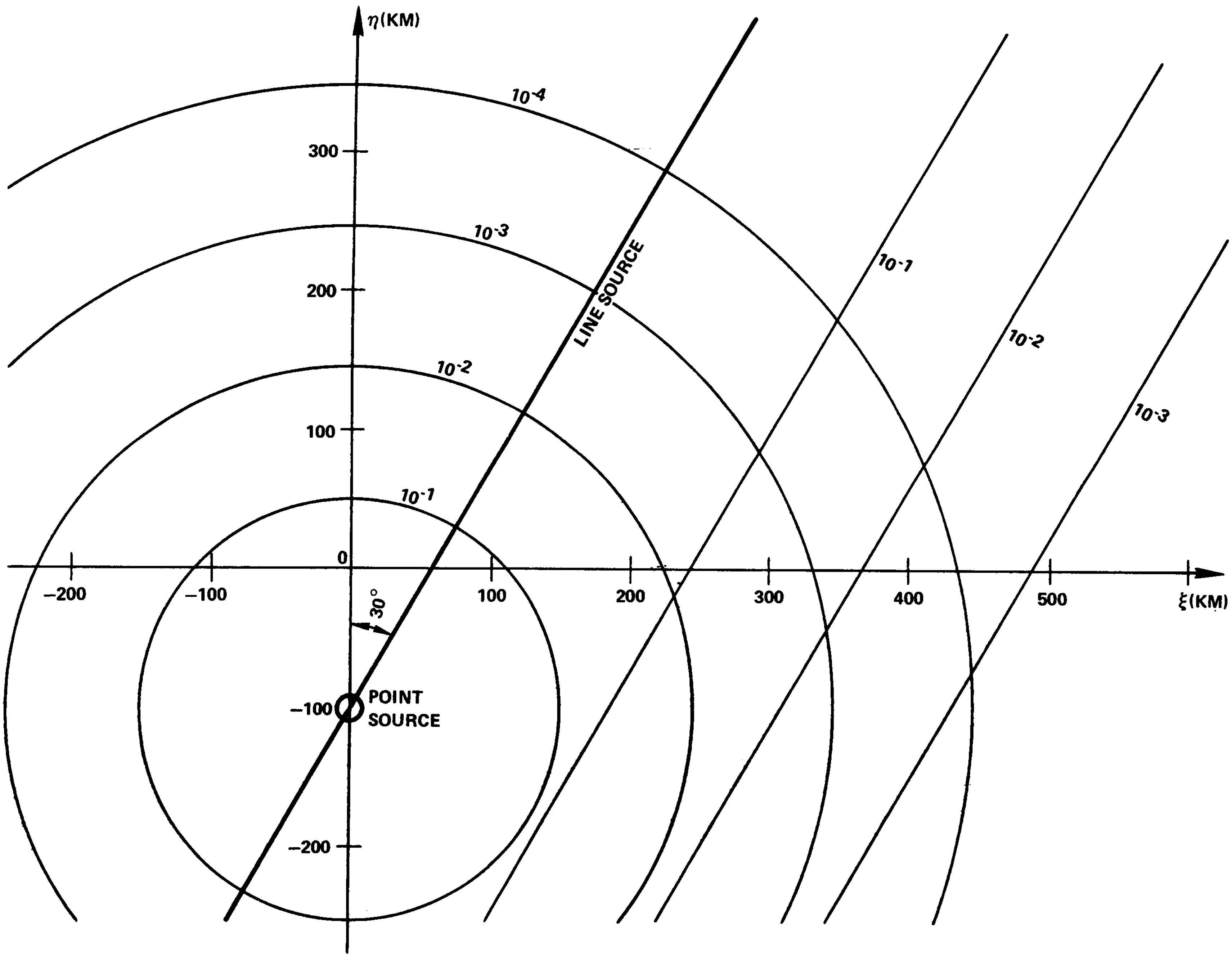

FIGURE 6 - CONTOUR MAP OF CONSTANT DENSITY PROFILES DUE TO A POINT AND A LINE SOURCE AT Z = 110 KM IN THE CASE OF H = 25 KM

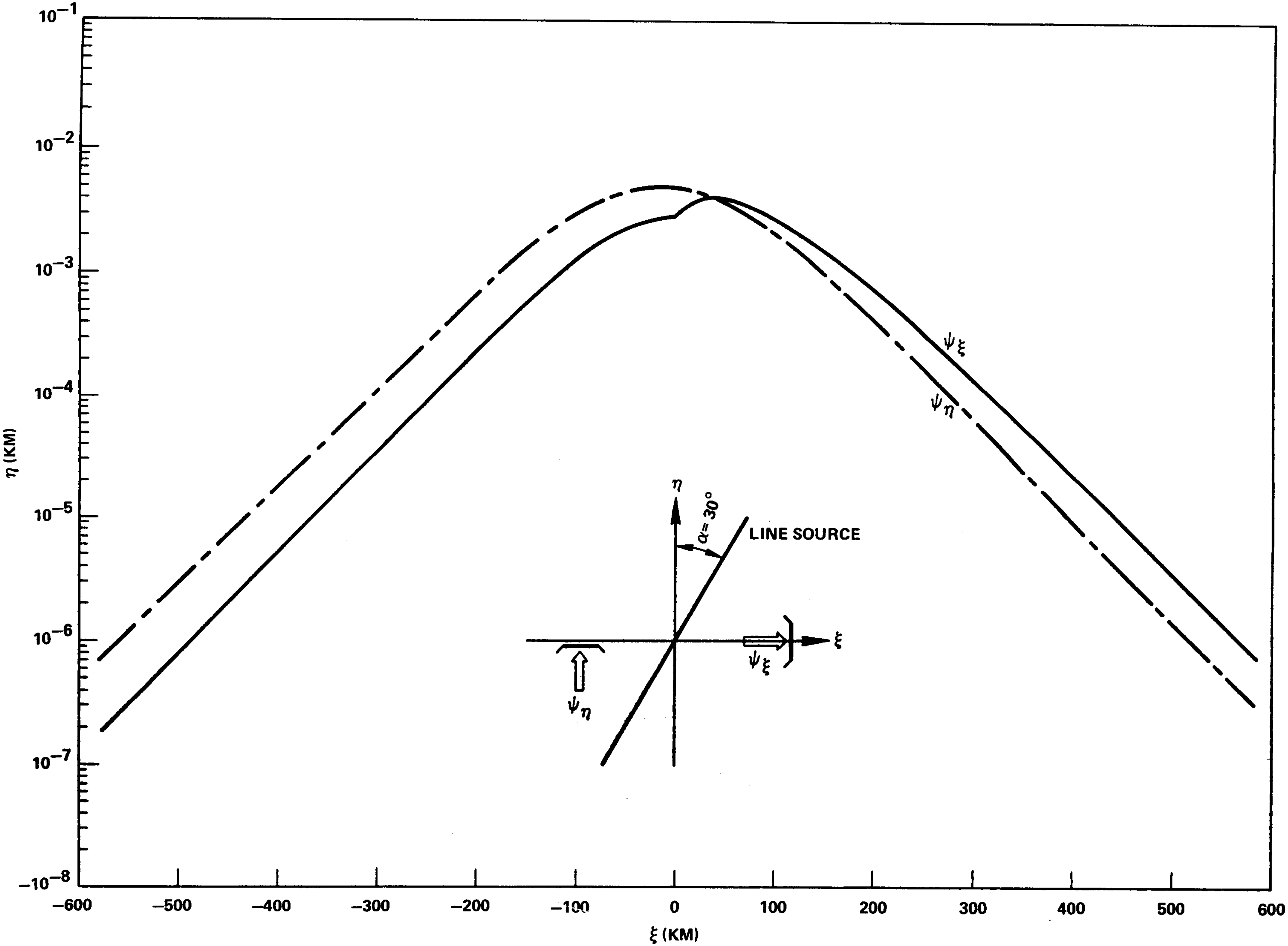

FIGURE 7 - ξ AND η – FLUX VARIATIONS VS ξ AT Z = 110 KM IN THE CASE OF AN OBLIQUE LINE SOURCE 30° WITH THE η – AXIS AND H = 25 KM