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TITLE- Performance of Error Detecting Coding on NASCOM Data Circuits

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ABSTRACT

The Apollo Command System provides a means of transmitting data and other information necessary for command purposes from the Manned Spacecraft Center in Houston to the Apollo Command/Service Module, the Lunar Module or the Saturn Launch Vehicle. The terrestrial communication links in the Apollo Command System employ error-detection and retransmission for error control. This report uses the code spectrum to estimate the probability of undetected error in the terrestrial portion of the system. The probability of detected error and retransmission is also considered.

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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

The Apollo Command System provides a means of transmitting data and other information necessary for command purposes from the Manned Spacecraft Center (MSC) in Houston to the Apollo Command/Service Module, the Lunar Module or the Saturn Launch Vehicle. Command data in the form of command loads and execute command requests (ECR) are sent from the Mission Control Center-Houston (MCC-H) located at MSC to the Goddard Space Flight Center (GSFC) in Greenbelt, Maryland, over wide band data lines at 50.0 kbps* and are then distributed to remote sites over high speed data lines at 2.4 kbps. The signal processing which results at the remote site and any subsequent transmission to the spacecraft depend on the type of command data. On both the MCC-H-to-GSFC link and the GSFC-to-remote site link, the command data is protected by a Bose-Chaudhuri-Hocquenghem (BCH) error detecting code. If no error is detected in the command data received at the remote site a validation signal called a Command Analysis Pattern (CAP) is sent through GSFC back to MCC-H over the same type of circuits used for the command data. This CAP is itself protected by BCH error detecting codes.

The purpose of this report is to estimate the probability of an undetected word error in reception of command data at the remote site or in reception of a CAP at MCC-H. The probability of a word containing an error which is detected is also considered.

1.1 FORMATS

The format of command data is basically a 60 bit sequence of binary symbols called a subblock. The subblock consists of 30 data bits, 27 parity check bits, and 3 filler bits. This format is used from MCC-H, through GSFC to the remote site. In addition, on the link from MCC-H to GSFC, the subblocks are formatted into a 600 bit block. The 600 bit block contains 8 subblocks and 120 bits of control information including 33 parity

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*The present rate is 40.8 kbps. It will be increased to 50.0 kbps for AS-205 and subsequent missions.

check bits. If command data passes through the GSFC and the 33 parity bits do not indicate an error, then the 120 bits of control information are removed and the data in the form of subblocks is transmitted to the remote site.

The CAP indicating validation of the command data received at the remote site is sent back to MSC in a 40 bit subblock which includes 14 parity check bits. Again, this subblock becomes part of a 600 bit block on the link from GSFC to MSC.

An error in the command data may be detected at GSFC, in which case the data is not forwarded to the remote site, or an error may be detected at the remote site. In either case, no CAP will be returned to MCC-H and retransmission will occur.

For a more detailed description of the ground communication links and the signal processing see Reference (1).

1.2 Performance Specifications

The Communications Division of GSFC performs periodic checks on the bit error rates of the high speed data channels to the remote sites and the wide band data channels to MSC. These checks consist of counting the errors which occur in a known pseudo-random sequence transmitted over a single channel. The error count is tabulated every thirty minutes over a 24 hour period. If the error rate exceeds one in 10^5 on either a high speed or a wide band link corrective action is taken. Error rates less than one in 10^5 are considered acceptable. No more detailed statistics, such as distribution of error burst lengths are taken[(2)].

The highest rate of undetected command data word error which is considered acceptable on the overall transmission between MCC-H and the spacecraft is one in 10^9. This implies that the ground network should perform at least this well by itself.

1.3 Analysis

There are several ways to estimate the effectiveness of an error detecting code used on a particular channel. Simulation is the most straightforward way. Either using the particular channel in real time or using a recording of its error pattern for a given interval of time, the number of detected and undetected word errors can be tabulated. This has been done for some particular channels and particular codes[(3,4)].

Alternatively, the channel can be mathematically modeled and the performance of the code can be calculated directly based on the parameters of the model[(5)].

The approach used in this report lies between the two above extremes. The probability, P(m,n), that m errors occur in a block of n bits is used according to a method developed by Elliot[3].

The only data available on NASCOM circuit errors is the overall error rate. By itself, this can be used to obtain a bound on the probability of undetected error, but the bound is too loose to be of any value. The values of P(m,n) used in the analysis of this report come from the work of Townsend and Watts[4]. It is felt, for reasons given below, that values of P(m,n) from the NASCOM circuits would yield results very nearly the same.

Measurements of probability of detected error have been made and are reported below; a simple bound on probability of detected error based on overall bit error rate is given.

2.0 ANALYSIS OF CODES

The BCH codes used in the transmission of Apollo command data are (n,k) group codes, i.e., each code word consists of k bits which can be specified arbitrarily followed by n-k bits, each of which is a linear combination of the first k bits. The first k bits which can assume the value of any of the 2^k possible k bit binary sequences are called information bits; the following n-k bits are called parity check bits. If any of the received parity check bits fail to equal the appropriate linear combination of received information bits, an error is claimed to have been made.

The error detection capabilities of BCH codes and the implementation of coders and decoders is discussed in detail in Reference (6).

2.1 Truncated Codes

The most common form of BCH codes have the length n equal to

$$n = 2^m - 1 \quad , \quad m = 3,4,5 \ldots$$

For reasons of formatting it may be desirable to use codes of arbitrary length. The length of BCH codes may be varied arbitrarily by truncating. In operation, the first i information bits of each code word are assumed zero and not transmitted. At the decoder, the received n-i bit code word is prefixed by a sequence of i zeros and then decoded as usual. The result, in effect, is an (n-i, k-i) linear code which can detect any pattern of errors the original (n,k) code could detect. The probability of undetected error with the truncated code is generally less than with the original code, but there is corresponding decrease in information rate[7].

In the Apollo command data formats the (600, 567) code is a truncated (1023, 990) code, the (57, 30) is a truncated (63, 36), and the (40, 26) is a truncated (127, 113).

2.2 Probability of Undetected Error

Elliot[3] characterizes a channel by the probability, P(m,n), that m errors occur in a block of n transmitted bits. The corresponding parameter of a code is its spectrum, ω(m), the average number of code words at Hamming distance m from a typical code word. Given this information the approximate probability of undetected error is

$$\bar{P}_u = \sum_{m=1}^{n} \omega(m) \frac{P(m,n)}{\binom{n}{m}} \tag{1}$$

Furthermore, if the spectrum of the code is not known, or if the code is so large that the spectrum cannot be easily computed, an ensemble average over all (n,k) codes yields the average spectrum,

$$\bar{\omega}(m) = \begin{cases} 2^{-(n-k)}\left[\binom{n}{m} - \binom{n-k}{m}\right], & 0 < m \le n-k \\ 2^{-(n-k)}\binom{n}{m}, & n-k < m \le n \end{cases} \tag{2}$$

If the minimum Hamming distance is known to be D, Equation (1) becomes

$$\bar{P}_u \cong \sum_{m=D}^{n} \bar{\omega}(m) \frac{P(m,n)}{\binom{n}{m}} \tag{3}$$

The derivation of $\bar{P}_u$ and $\bar{\omega}(m)$ which are due to Elliot[3] are given in Appendices A.1 and A.2, respectively.

Elliot's analysis applies to any group code. It is shown in Appendix A.3 that a truncated group code is itself a group code and therefore Equation (3) is applicable to the truncated codes used in the NASCOM network.

2.3 Evaluation of Estimate of Probability of Undetected Error

The approximate spectrum of a code can be easily calculated from Equation (2). Values of P(m,n), however, are not available for either the 2.4 kbps or the 50.0 kbps NASCOM channels. The values for P(m,n) used in the following analysis were obtained by Townsend and Watts(4) on 2.0 kbps lines of various lengths. The Townsend and Watts data had an overall error rate of 3.2×10^{-5}. There is some justification for using this data. Alexander, Gryb, and Nast(3) obtained error statistics at 600 and 1200 bps over telephone channels. The overall error rate, the distribution of error rates among calls, and P(m,n) agree closely between the Alexander, Gryb, and Nast data and the Townsend and Watts data(4,9). It is felt, therefore, that error statistics on telephone channels are not strongly affected by the particular speed, modem or set of lines under consideration.

A further assumption is that the presence of satellite communication links in some of the Apollo command data channels between GSFC and remote sites does not appreciably alter P(m,n).

3.0 PERFORMANCE OF CODES

From the values of P(m,31) and P(m,63) determined by Townsend and Watts(4), the values of P(m,57) and P(m,40) were interpolated. The approximate code spectrum from Equation (2) was used in the expression for probability of undetected error, Equation (1). The results were,

$$(57,\ 30)\ \text{code:}\quad \overline{P}_u = 2.1 \times 10^{-13} \qquad (4)$$

$$(40,\ 26)\ \text{code:}\quad \overline{P}_u = 2.2 \times 10^{-9} \qquad (5)$$

The difference of four orders of magnitude in the $\overline{P}_u$ is reasonable because of the shorter length and smaller redundancy of the (40, 26) code(3).

Of course, the values of $\overline{P}_u$ shown in Equations (4) and (5) apply only to transmissions between GSFC and remote sites. Exact analysis of the 600 bit code on the 50.0 kbps line between GSFC and MCC-H was not performed because P(m,600) could not be accurately extrapolated from the data of Townsend and Watts. It does not appear that the overall values of $\overline{P}_u$ would be more than twice those values in Equations (4) and (5). The 50.0 kbps line has an average error rate at least as good as 10^{-5} and it should have burst characteristics somewhat similar to the other telephone channels. If the 33 parity check bits in the 600 bit block were not checked, then, effectively, the 60 bit block would be exposed to errors for which P(m,n) would be twice that given by the Townsend and Watts data. The fact the 600 bit code will detect the vast majority of error patterns which can corrupt it means that the overall values of $\overline{P}_u$ are much less than twice the values given in Equations (4) and (5).

3.1 Probability of Detected Error

The probability of detected error is of interest in evaluating the NASCOM system. The rate of 600 bit block detected errors was measured during the AS-501 mission. The average probability of detected error in 600 bit words was 5.06×10^{-4}. This average was based on an eight hour period of observation.

No measurements of a similar nature were performed on the 2.4 kbps lines. However a simple upper bound can be stated. If the errors on the 2.4 kbps line were independent and scattered, then any subblock containing an error would most likely contain only a single error. This is the extreme case and determines an upper bound on the probability of detected subblock error:

$$P_d \leq 60 \times 1 \times 10^{-5} = 6 \times 10^{-4} \quad (6)$$

The probability of detected error per 600 bit block is an upper bound to the probability of detected error per 60 bit subblock. Therefore, a bound on the overall probability of detected error per 60 bit subblock between MCC-H and a remote site is

$$P_d \leq 5 \times 10^{-4} + 6 \times 10^{-4} = 1.1 \times 10^{-3} \quad (7)$$

The corresponding bound for the 40 bit CAP transmitted from the remote site back to MCC-H is

$$P_d \leq 5 \times 10^{-4} + 4 \times 10^{-4} = 9 \times 10^{-4} \quad (8)$$

3.2 Discussion

The use of the Townsend and Watts data seems reasonable in view of the fact that essentially no data from NASCOM circuits is available beyond just the overall error rate.

There may be a question in the reader's mind concerning the accuracy of Elliot's expression for $\overline{P}_u$, Equation (3), since it does not require knowledge of the distribution of burst error lengths. Elliot simulated the use of a BCH (31, 21) error detecting code using the Alexander, Gryb, and Nast data. The simulation resulted in a probability of error 20% larger than that estimated by Equation (1) using P(m,n). According to Elliot[10], the results of Equation (3) also agree with simulation to within 20%. Thus, although the derivation of Equations (1) and (3) (see Appendices A1 and A2) may not be intuitive, the results are reasonably accurate.

In its present state, the Apollo command system works well. There are, however, two alternatives which might be considered. First, the redundancy in the 60 bit subblock might be reduced by removing check bits; this would force $\overline{P}_u$ closer to the 10^{-9} limit. Clearly, from the performance of the (40, 26) code, the command data code word could be no shorter than (40, 30). Thus a gain of at most 50% in information rate might be realized.

Second, forward acting error correction might be incorporated. Pure error correction is probably not desirable because of the burst characteristics of the channel. Codes redundant enough to correct almost all anticipated error burst patterns would result in a very low information rate. A hybrid error detection and correction system would be better than a pure correction system. The hybrid code could, for example, correct all single and double errors, and detect the majority of the long burst error patterns. The exact gain in information throughout rate possible with hybrid coding would require further analysis.

It is instructive to consider the effect of reducing the redundancy in the 60 bit subblock on the speed of operation of the Apollo Command System. Command loads are sent to remote sites and stored well before their intended use. Thus the information rate of the command loads has no effect on the speed of operation of the overall system. On the other hand, an execute command request is executed immediately upon receipt at the remote site. An analysis of the delay times involved[1] has shown that even by removing all redundancy from the 60 bit ECR subblock the delay between commanding the ECR at MCC-H and its execution at the remote site would be reduced by no more than 10%.

In conclusion, the error detecting coding in the Apollo Command System ground network offers the desired protection against undetected error with an acceptable cost in time delay.

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Attachment
(Appendices 1-3)

APPENDIX A1

Probability of Undetected Error

Let x represent a code word in the code C. Let y represent any received word. When using an error detecting code, an undetected error occurs if the received word, y, is a code word different from the transmitted code word, x. Thus the probability of an undetected error when x is transmitted is

$$\sum_{y(\neq x)\,\varepsilon C} P(x\to y) \qquad \text{(A1)}$$

where $P(x\to y)$ is the probability that the channel changes the word x to a word y. Assuming the code words are used with equal frequencies, the average of the above probability is

$$P_u = \frac{1}{N}\sum_{x\varepsilon C}\ \sum_{y(\neq x)\varepsilon C} P(x\to y) \qquad \text{(A2)}$$

where N is the number of code words in the code. Thus P_u is the probability of undetected error when using a specific code C on a specific channel.

Now assume that the channel is known to be symmetric and to have specified values of P(m,n), the probability of m errors in a block of n symbols, but that no more detailed statistics are known. Symmetry implies

$$P(x\to y) = F(Z) \qquad \text{(A3)}$$

where

$$Z = y-x \quad (\text{mod } 2) \qquad \text{(A4)}$$

Since $P(x\to y)$ cannot be calculated for this channel, P_u will be averaged over all the permutations of bit positions in each block of n bits. Let π denote a permutation of n bits.

$$\overline{P}_u = \frac{1}{n!}\sum_{\pi} P_u = \frac{1}{n!}\sum_{\pi}\frac{1}{N}\sum_{x\varepsilon C}\ \sum_{y(\neq x)\,\varepsilon C} P(\pi x\to\pi y)$$

$$= \frac{1}{N}\sum_{x\varepsilon C}\ \sum_{y(\neq x)\varepsilon C}\frac{1}{n!}\sum_{\pi} P(\pi x\to\pi y) \qquad \text{(A5)}$$

$$= \frac{1}{N}\sum_{x\varepsilon C}\ \sum_{y(\neq x)\varepsilon C}\frac{1}{n!}\sum_{\pi} F(\pi Z)$$

Any n-place binary sequence containing exactly m ones is left invarient by m!(n-m)! permutations of its digits. Therefore, if Z contains m ones,

$$\frac{1}{n!}\sum_{\pi} F(\pi Z) = \frac{m!(n-m)!}{n!}\sum_{u} F(u) = \frac{P(m,n)}{\binom{n}{m}} \qquad (A5)$$

where u is any n-place binary sequence with exactly m ones. Thus,

$$\bar{P}_u = \sum_{m=1}^{n} \omega(m) \frac{P(m,n)}{\binom{n}{m}} \qquad (A6)$$

where $\omega(m)$ is the average number of code words at a distance m from a typical code word.

APPENDIX A2

Average Code Spectrum

It is known[6] that there are $2^{k(n-k)}$ distinct group codes of block length n and dimension k. Also, a given binary word, Z, of length n will belong to $2^{(k-1)(n-k)}$ of those codes provided the information portion of Z does not contain only 0's. Now, there are

$$\left[\binom{n}{m} - \binom{n-k}{m}\right] \tag{A7}$$

binary words of weight m having nonzero information parts whenever $0<m\leq n-k$, and there are $\binom{n}{m}$ such words whenever $n-k<m\leq n$. Therefore, the average number $\overline{\omega}(m)$ of code words of weight m is

$$\begin{aligned} \overline{\omega}(m) &= \frac{1}{2^{k(n-k)}}\left[\binom{n}{m} - \binom{n-k}{m}\right] 2^{(k-1)(n-k)} \\ &= \frac{1}{2^{(n-k)}}\left[\binom{n}{m} - \binom{n-k}{m}\right] \end{aligned} \tag{A8}$$

when

$$0 < m \leq n - k$$

and

$$\overline{\omega}(m) = \frac{1}{2^{(n-k)}}\binom{n}{m} \tag{A9}$$

when

$$n - k < m \leq n$$

This average is over all group codes of block length n and dimension k.

If the minimum distance of a code is D, the probability of undetected error, Equation (A6), can be written:

$$\overline{P}_n \cong \sum_{m=D}^{n} \overline{\omega}(m) \frac{P(m,n)}{\binom{n}{m}} \tag{A10}$$

APPENDIX A3

Group Property of Truncated Code

Any code is a group code if it satisfies the four axioms which define a group: closure, identity, inverse, and associativity.

Denote by c the original code and by c_T the truncated code. Let x and y be code words in c and x' and y' the truncated versions of those code words.

To show closure, assume x' and y' are in c_T. The generating code words x and y must have zeros in the first i positions. Thus the code word x + y (mod 2) must have zeros in the first i positions. Therefore, x' + y' is in c_T.

Since the zero vector in c has zeros in the first i positions, c_T will contain an all zero vector which provides an identity.

Any code word in c_T is its own inverse.

The code c_T is clearly associative. Therefore c_T is a group code.

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